AP Calculus AB Syllabus. Kelli Gamez Warble 2009

Course Overiew

COURSE FOCUS:

▪the study of functions in a variety of representations (numerical, graphical, analytical, and verbal);
▪the investigation of limits formally and conceptually;
▪the evaluation of derivatives and integrals graphically, numerically and analytically;
▪the connection of differential and integral calculus to geometry, economics and mechanics;
▪using the computer or graphing calculator as a tool for understanding and solving mathematical problems;
▪using and explaining the Fundamental Theorem of Calculus.

STUDENT ASSESSMENT:

Students will be assessed using a variety of test questions, quizzes, assignments, and projects. Some tests will be cumulative. Where appropriate, programmable calculators, calculator-based laboratories, and computers will be used.

TEXTBOOK:

Calculus With Analytic Geometry by Roland Larson, Robert Hostetler, and Bruce Edwards; 6th edition, 1998, Houghton ;Mifflin Company.

TECHNOLOGY RESOURCES:

Students are frequently required to use a graphing calculator (most commonly the TI-84 or TI-89) to complete investigations into the nature of functions and the conceptual framework of Calculus. In addition, the classroom is equipped with Vernier LabPros and the electronic probes accompanying these. The LabPros are connected to the TI graphing calculators to perform laboratory investigations demonstrating the real-world use of Calculus concepts.

The most commonly used technology resource books are listed below:

Exploring Calculus with a Graphing Calculator by Charlene Backmann and Theodore Sundstrom; ©1992 by Addison-Wesley Publishing Company, Inc.

Calculus with a TI-83 Graphing Calculator by George Best and Sally Fischbeck; ©Venture Publishing.

Real World Math with a CBL System by Chris Brueningsen, Bill Bower, Linda Antinone, and Elisa Brueningsen; ©Texas Instruments Incorporated.

Modeling Physics Instructional Materials edited by Larry Dukerich; ©Modeling Instruction Program from Arizona State University.

TARGET AREA PROJECTS:

Students will be required to complete several projects that focus on collecting, analyzing, and reporting results of laboratory-based mathematical investigations. Results will be reported using *multiple representations,* including a numerical table of data, a graph of the data, a mathematical model that fits the data, and a written explanation of the relations among these representations. Students will be required to give whiteboard presentations of the results of their investigations to their classmates. These whiteboarding sessions will encourage a discussion of the similarities and differences among the results of different groups. The investigations may be chosen from among the following:

- exploring real-world relationships that generate linear data, investigate slope and intercept and generate a model function;
- exploring families of functions by investigating alteration of parameters A, B, C, and D when g(x) = A f(Bx + C) + D;
- position, velocity, and acceleration data of a swinging pendulum;
- position, velocity, and acceleration data for a falling object;
- derivatives of sinusoids as related to an oscillating spring;
- approximation of area under a velocity vs. time graph using rectangles;
- exponential decay curves as modeled by a population of m & m's;
- temperature curves and Newton's Law of Cooling;
- using a computer to calculate area under a force vs. displacement graph to model work done.

Course Planner

PREPARATION FOR CALCULUS (Chapter P) 2 weeks

Overview: The student will be introduced to all of the elementary functions to be used throughout the course. The unit will thus be a review of topics from previous courses but through a new approach consisting of extensive use of graphical, numerical, and verbal modeling of functions.

Goals: The student will:

1. be introduced to the TI-83/89 graphing calculator, including graphing, function evaluation, editing equations, and using lists and tables.
2. graph and interpret functions and data; focusing on intercepts, symmetry, and points of intersection.
3. review slope (rate of change) and general equations of linear functions.
4. LAB: collect real-world data which generates a linear relationship, graph and fit a model to the data (e.g. ball bouncing, penny mass, rubber band shoot, et cetera).
5. GRAPHING CALCULATOR LAB: explore families of functions by investigating alteration of parameters A, B, C, and D when g(x) = A f(Bx + C) + D.
6. review functions and function notation, domain and range, transformations, and classification and combinations of functions.
7. fit linear, quadratic, and trigonometric models to data.
8. LAB: collect and analyze sinusoidal data of a swinging pendulum with motion detector.

LIMITS AND THEIR PROPERTIES (Chapter 1) 2 weeks

Overview: The student will be given a conceptual understanding of the idea of a limit and the significance of the existence (or nonexistence) of a limit to a function.

Goals: The student will:

1. be introduced to limits by examining graphs, tables, and algebraic representations of functions which may be undefined at a certain point, but nevertheless approach a certain value.
2. be introduced to functions which have no limit at key values in their domain, examine such functions graphically, tabularly, and analytically.
3. study strategies for finding limits analytically.
4. investigate continuity of functions and its connection to the existence of limits.
5. investigate asymptotes and their relation to limits of functions.

DIFFERENTION (Chapter 2) 4 weeks

Overview: The student will be given a practical understanding of the definition of the derivative and its interpretation as an instantaneous rate of change graphically, numerically, algebraically, and verbally. The student will be introduced to derivative formulas for all of the functions covered in Chapter P, as well as to the rules for differentiating products, quotients, and composite functions.

Goals: The student will:

1. LAB INVESTIGATION: collect real world motion data using a Sonic Ranger, relate position and velocity graphs generated.
2. distinguish between average and instantaneous rate of change on a position vs. time graph to define velocity.
3. GRAPHING CALCULATOR LAB: use technology to zoom in and estimate slope of a curve over a very small interval.
4. discuss difference quotient and the average rate of change of a function.
5. define derivative using the limit definition, be able to interpret the derivative function graphically, numerically, and from a formula.
6. derive the general power rule for derivative formulas; find derivatives of power functions, polynomials and trigonometric functions.
7. LAB INVESTIGATION: Answer the question, “Is the derivative of the product the same as the product of the derivative?”; use results to generate the product rule.
8. use product and quotient rules to find first and higher order derivatives.
9. use the chain rule to find derivatives of compositions of simpler functions.
10. study implicit differentiation as another application of the chain rule.
11. find related rates of change and solve problems with related rates.

APPLICATIONS OF DIFFERENTIATION (Chapter 3) 5 weeks

Overview: The student will use the derivative in solving problems, including optimization and graphing.

Goals: The student will:

1. GRAPHING CALCULATOR LAB: define critical numbers, use a graphing calculator to relate critical numbers to maximums and minimums on function.
2. use the first derivative to identify relative extrema and whether a function is increasing or decreasing,
3. LAB: use a motion detector and computer to collect and analyze data for an object undergoing constant velocity, constant acceleration, and simple harmonic motion; generate graphs of and relate position, velocity and acceleration to the first and second derivative functions.
4. use the second derivative to identify inflection points and regions of concavity.
5. investigate the behavior of functions as the independent variable approaches infinity to find horizontal asymptotes (if any).
6. use the concepts of intercepts, symmetry, domain and range, continuity, asymptotes, differentiability, extrema and concavity to analyze and sketch functions.
7. learn to optimize functions; use the first and second derivative to find maximums and minimums of real-world applications; find global extrema.
8. use Newton's Method to approximate roots of a function; identify problems arising when using Newton's Method.
9. estimate a graph as being close to the tangent line on a small scale (linear approximations); use differentials to estimate incremental changes in the dependent variable with each incremental change in the independent variable.
10. use differentiation in qualitative, graphical reasoning; represent economic ideas graphically, including cost, revenue, profit, and marginality.

INTEGRATION (Chapter 4) 5 weeks

Overview: The student will utilize the idea of subdividing a quantity to produce Riemann sums which, in the limit, will yield a definite integral. The student will be given a practical understanding of the definite integral as a limit of Riemann sums and understand the connection between the derivative and the definite integral in the Fundamental Theorem of Calculus. The student will learn to go backward from a derivative to the original function, first graphically and numerically, then analytically. The student will be introduced to the concept of a differential equation.

Goals: The student will:

1. find simple anti-derivatives and be introduced to the concepts of a differential equation including a parameterized family of solutions and the use of initial conditions to establish a unique solution.
2. utilize indefinite integral notation; find basic anti-derivatives; find anti-derivatives of sums, differences and constant multiples of functions.

3. GRAPHING CALCULATOR LAB: program a graphing calculator to draw slope fields, use slope fields to solve differential equations.
4. understand integral notation, use Riemann sums to estimate area/definite integral.
5. GRAPHING CALCULATOR LAB: Program a graphing calculator to do Riemann sums; use a graphing calculator to sum more and more rectangles, relate this to the exact area under a curve.
6. given rate of change of a function, find original amount by calculating area; interpret the units and meaning of definite integrals; use definite integrals to solve application problems.
7. be introduced to the Fundamental Theorem of Calculus with emphasis upon the connection of the definite integral to *overall change* in the original function.
8. learn to perform simple substitution in definite integrals where only a constant factor must be introduced; utilize less obvious substitutions in definite integrals.
9. compare the effectiveness of the Left, Right, Midpoint and Trapezoid rules to estimate integrals numerically.
10. analyze errors in integral approximations; use Simpson's rule and recognize it as the most accurate estimate.

LOGARITHMIC, EXPONENTIAL, AND OTHER TRANSCENDENTAL FUNCTIONS (Chapter 5) 4 weeks

Overview: The student will review the concept of the inverse of a function and find function inverses graphically, numerically, and analytically. The student will find derivatives and anti-derivatives of various inverse functions, and utilize them to solve differential equations. The student will be introduced to hyperbolic functions.

Goals: The student will:
1. review exponential functions.
2. LAB: collect real-world exponential data by looking at the decay of m & m populations.
3. define inverse functions graphically, algebraically, numerically, and in words.
4. be introduced to logarithmic functions, define them as the inverse of exponential functions.
5. define the number *e* and natural logs.
6. GRAPHING CALCULATOR LAB: Draw tangents to exponential functions and look for patterns to derive formulas for derivatives of e^{kx} and b^x .
7. use the chain rule to find derivatives of logarithmic functions when the derivative of exponential functions is known.
8. practice finding the derivatives of exponential and logarithmic functions.
9. use separation of variables to solve certain special first-order differential equations analytically.
10. generate solutions of the exponential growth and decay equations; apply these solutions to compound interest, radioactive decay, and Newton’s Law of Cooling.
11. LAB: collect and analyze real-world data relating to temperature curves; relate this to the solution of differential equations involving exponential functions and logarithms.

12. review inverse trigonometric functions and use the chain rule to derive formulas for the derivatives and anti-derivative of inverse trigonometric functions.
13. define and describe properties of hyperbolic functions; find derivatives and anti-derivatives of hyperbolic functions.

APPLICATIONS OF INTEGRATION (Chapter 6) 3 weeks

Overview: The student will apply the definite integral to uses in geometry and physics.

OBJECTIVES:

The student will:

1. use integrals to calculate the area between two or more curves.
2. use integrals to calculate volumes of solids of revolution by slicing (disk method) and the nested cylinder method.
3. use integrals to calculate arc length and surface area of geometric shapes.
4. calculate center of mass of objects with non-uniform density by dividing up regions where density varies.
5. learn the definition of work and use definite integrals to calculate work done in various physical situations.

INTEGRATION TECHNIQUES, L'HOPITAL'S RULE, AND IMPROPER INTEGRALS (Chapter 7) 3 weeks

Overview: The student will learn several techniques of integration and how to utilize the table of integrals. The student will be introduced to L'Hopital's Rule for finding limits and Improper Integrals.

OBJECTIVES:

The student will:

1. perform the integration by parts method for indefinite and definite integrals.
2. recognize a given integral as a standard form in a table; perform substitution to convert a given integral into a standard form to evaluate integrals
3. use reduction formulas and partial fractions to perform integration.
4. use L'Hopital's rule to evaluate limits with indeterminate form.
5. develop a graphical and numerical understanding of the convergence and divergence of improper integrals.
6. learn to perform a comparison test for improper integrals.