PHS 542: Integrated Mathematics and Physics (3 semester hours)

Instructor: Robert Rowley, M.S. in Physics. Bob has 60 graduate credits in physics at ASU. He taught physics, calculus, programming, and digital electronics for 25 years. He developed the **Primer on Geometric Algebra for introductory mathematics and physics** with David Hestenes, inventor of geometric algebra and founder of the ASU MNS program.

This class will meet at ASU-Tempe campus in the Physical Sciences Center, room H-355, from 4:00pm to 6:30pm MTWR for 5 weeks from July 2 to Aug. 2, 2018. No class on Wed. July 4.

Catalog description: Mathematical models and modeling as an integrating theme for secondary mathematics and physics. Enrollment by teams of mathematics and physics teachers encouraged.

Course purpose:
Science is about modeling the natural world (or, modeling our observations of the natural world). Math is a set of formal tools for making our models clear and precise, and for working out the consequences with rigor. We all make internal mental models of the natural world, but without formalizing them (with the use of math) they tend to be ad hoc, vague, unreliable stories.

Math, on the other hand, has precision and clarity of structure, but without being applied to meaningful models it tends toward being a meaningless proliferation of empty rules. (Caveat: those tendencies are just that—tendencies, not absolutes.)

Science and math both become more vibrant, fecund, and rewarding when joined together, and that's true of education as well as professional practice. Thus, we want to integrate them all throughout the instructional process.

Course objectives:
- learn ways to coordinate algebra and pre-calculus with physics by exploring models that are common to math and physics.
- develop student activities that support math-physics coordination.
- establish a common language and set of representational tools that math and physics teachers can use with their students.

Course content:
Abstract mathematical concepts such as *variable*, *function* and *rate* will be used to develop mathematical models of physical situations. Emphasis will be placed on use of technology, which de-emphasizes the process of data-gathering, and shifts the focus to data interpretation, model identification and generalization.

Geometric algebra will be introduced. Geometric algebra (GA) is a unified mathematical language for the whole of physics. GA is suitable for a first year physics course (at advanced high school and post-secondary levels). Participants will learn how to formulate, analyze and apply basic math models for Newtonian mechanics without introducing coordinates. Teachers will learn GA using **the Primer on Geometric Algebra for introductory mathematics and physics**. Downloadable at http://geocalc.clas.asu.edu/html/IntroPrimerGeometricAlgebra.html

Expectations: Punctuality and active participation in class and group activities are crucial to learning. Homework will be assigned, but the emphasis will be on cooperative learning.

Suggested prior course: For physics teachers, a Modeling Workshop is recommended.

Basic mathematical models:

(1) Linear -- rate of change = constant ($\dot{Q} = k$)
graphs and equations for straight lines (i.e., velocity, acceleration, force, momentum, energy, etc.)

(2) Quadratic – change (in rate of change) = constant ($\ddot{Q} = k$)
graphs and equations for parabolas (accelerated motion, kinetic and elastic potential energy, etc.)

(3) Exponential - rate of change = proportional to amount ($\dot{Q} = kQ$)
graphs and equations of exponential growth and decay (population growth, radioactive decay, etc.)

(4) Harmonic -- change (in rate of change) = proportional to amount ($\ddot{Q} = kQ$, $k < 0$)
graphs and equations of trigonometric functions (waves and vibrations, harmonic oscillators, electricity and magnetism, etc.)

(5) Parametric and vector valued functions
graphs and parametric equations (vectors in 2 dimensions, uniform circular motion, the unit circle, projections from a vector point of view)

The theme of this course is to investigate these elementary functions as models of various physical and geometric phenomena, in one and two dimensions, with some emphasis on the idea that they are the simplest ways a change in one quantity relates to the change in another.

Why enroll in this course:

* Physics courses are typically weak in mathematical analysis of the models they develop. There is little time to analyze the functional properties that are identified, and rarely are they generalized to non-physics contexts. This course remedies that deficiency.

* Physics students learn to look at rate of change in a narrow kinematic context. This concept will be generalized and applied to a broader range of processes.

* Science is about discerning and representing structure. Mathematics has been called the "language of structure". Such a coincidence of interests should be exploited whenever possible.

* This course is a unique opportunity for math and physics teachers to amplify their effectiveness by cooperating to find a common language and set of representational tools for use with students. **Physics teachers: please alert your math teachers to this course, and invite them.**

WHY GEOMETRIC ALGEBRA? David Hestenes wrote:

Physics teachers are universally dismayed by the paltry understanding of mathematics that students bring from their mathematics courses. Blame is usually laid on faulty teaching. But I hold that the crux of the problem is deeply embedded in the curriculum. From the perspective of a practicing scientist, *the mathematics taught in high school and college is fragmented, out of date and inefficient*!

The central problem is found in high school geometry. Many schools are dropping the course as irrelevant. But that would be a terrible mistake for reasons already clear to Galileo at the dawn of science.

* Geometry is the starting place for physical science, the foundation for mathematical modeling in physics and engineering and for the science of measurement in the real world.
* Synthetic methods employed in the standard geometry course are centuries out of date; they are computationally and conceptually inferior to modern methods of analytic geometry, so they are only of marginal interest in real world applications.
* A reformulation of Euclidean geometry with modern vector methods centered on kinematics of particle and rigid body motions will simplify theorems and proofs, and vastly increase applicability to physics and engineering.

A basic pedagogical principle: *The depth and extent of student learning is critically dependent on the quality of the available mathematical tools.*

Therefore, we can expect a well-designed curriculum based on vector methods to produce significant improvements in the depth, breadth, and usefulness of student learning. …

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References for further study of GA:

D. Hestenes, "Oersted Medal Lecture 2002: Reforming the mathematical language of physics, " Am. J. Phys. **71:** 104-121 (2003). Get print copies from Jane Jackson, ASU Dept. of Physics.
D. Hestenes, *New Foundations for Classical Physics* (Kluwer, Dordrecht, 1986, 2nd ed. 1999)

Both are at David Hestenes' website: http://geocalc.clas.asu.edu