Speeder and Patrolman Problem

A speeder driving down the road at a constant 20 m/s, passes a patrolman parked on the roadside. The patrolman waits 3 seconds, then pursues the speeder, accelerating at a constant 4.0 m/s^2.

Q When the does the patrolman catch the speeder?

A Let t = time the patrolman is accelerating. Then t + 3 = time the speeder is traveling. At the instant the patrolman catches the speeder, the displacement, Δx, for both vehicles is the same.

$\Delta x = \bar{v}(t+3)$ speeder

$\Delta x = \frac{1}{2}at^2$ patrolman

$$\bar{v}(t+3) = \tfrac{1}{2}at^2$$

$$(20\,{}^{m}\!/_{s})\ (t+3) = \tfrac{1}{2}\left(4.0\,{}^{m}\!/_{s^2}\right)t^2 \text{ ignore units for a bit}$$

$$20t + 60 = 2t^2$$

$$2t^2 - 20t - 60 = 0$$

To solve this equation, use the quadratic formula (or your scientific calculator)

$$\frac{20 \pm \sqrt{400 + 480}}{4}$$

$$\frac{20 \pm 29.7}{4}$$

$$t = 12.4s$$

After the patrolman has accelerated 12.4s, he catches the speeder who has been traveling for 15.4s.

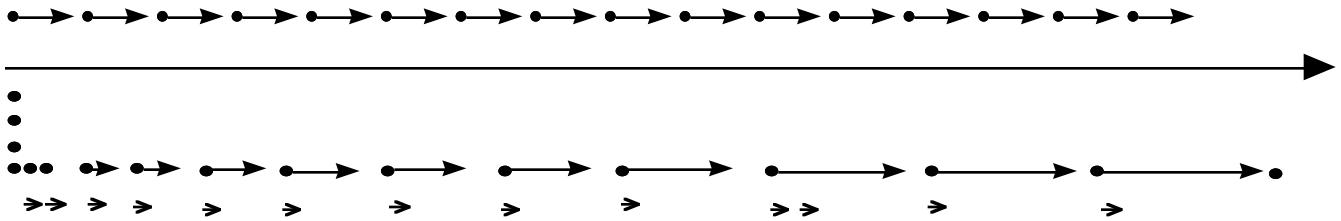
Q Where does the intercept occur? Keeping 3 sf's one can solve this with either equation:

A

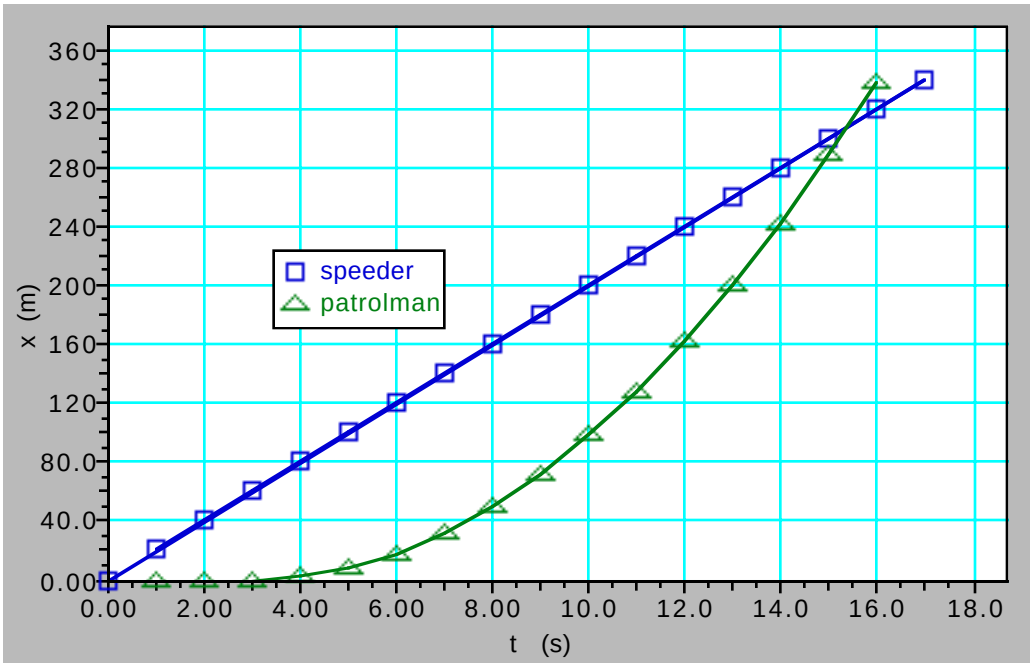
$\Delta x = 20\,{}^{m}\!/_{s}(15.4s)$ $\quad \Delta x = \frac{1}{2}\left(4.0\,{}^{m}\!/_{s^2}\right)(12.4s)^2$

$\Delta x = 308m$ $\quad \Delta x = 308m$

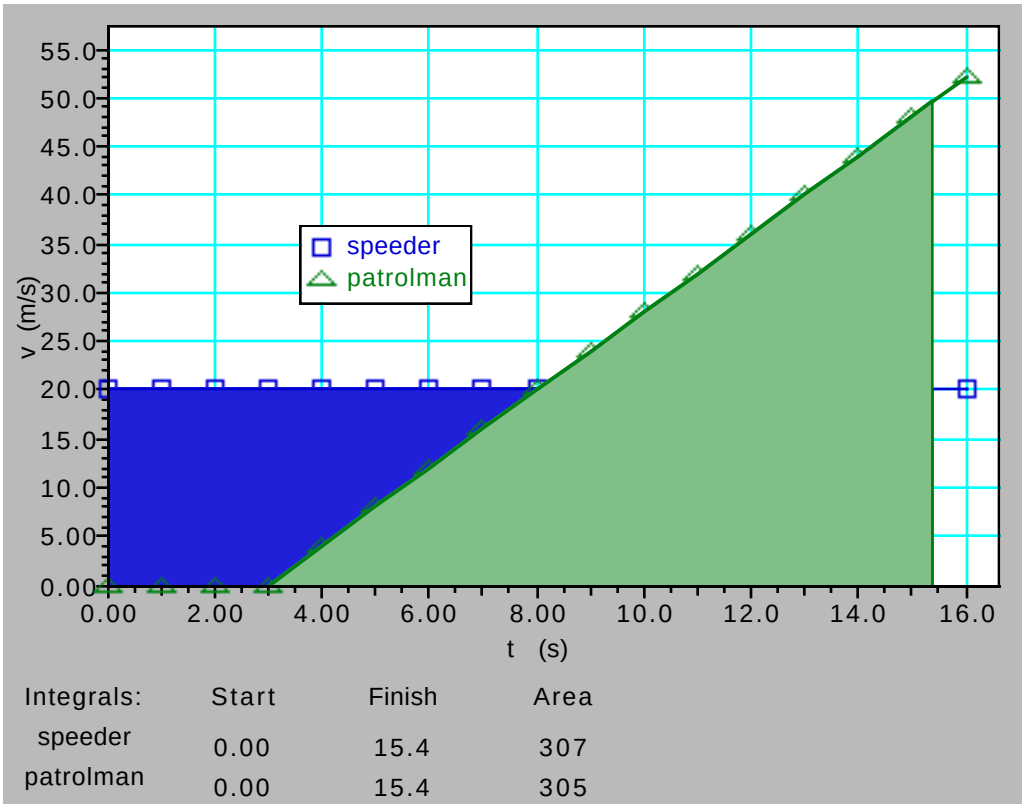
A motion map depicting this situation is shown below. The speeder's position is shown above the position axis, while the patrolman's position is shown below.



Below is a position vs time graph of the patrolman and speeder.



Below is a velocity vs time graph. The area under each curve has been shaded to show the approximate displacement of each vehicle.



Integrals:	Start	Finish	Area
speeder	0.00	15.4	307
patrolman	0.00	15.4	305