

UNIT VII - ENERGY (WITH LESS WORK)

Instructional Goals

1. View energy interactions in terms of transfer and storage

Develop concept of relationship among kinetic, potential & internal energy as modes of energy storage

emphasis on various tools (especially pie charts) to represent energy storage

apply conservation of energy to mechanical systems

2. Variable force of spring model (see lab notes: spring-stretching lab)

Interpret graphical models

area under curve on **F** vs **x** graph is defined as elastic energy stored in spring

Develop mathematical models

$$\mathbf{F} = k\mathbf{x}$$

$$E_{el} = \frac{1}{2}kx^2$$

3. Develop concept of working as energy transfer mechanism

Introduce conservation of energy

focus on $W = \Delta E$ in this unit

Working is the transfer of energy into or out of a system by means of an external force. The energy transferred, W is computed by $W = F_{\parallel} \cdot \Delta x$

the area under an F-x graph, where F is the force transferring energy.

Energy bar graphs and system schema represent the relationship between energy transfer and storage

4. Contrast conservative vs non-conservative forces

Energy transfers by conservative forces are reversible

5. Conservation of energy lab investigation - (see lab notes: 3 optional approaches)

6. Power (no specific labs)

Define power- rate at which energy is transferred: $P = \frac{W}{t}$

SI unit: watt

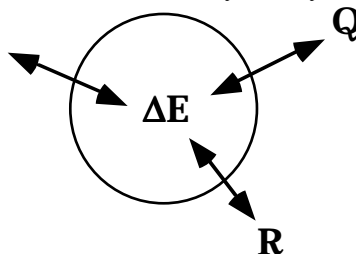
Overview

The traditional approach to teaching work and energy in a standard physics course often ends up being a rather imprecise, confusing mass of equations and definitions, such as "energy is the ability to do work" and $W = F\Delta x \cos\theta$. For example, the work-energy theorem is a point-particle model that is often inappropriately applied to situations that require the consideration of internal structure, and the 1st Law of Thermodynamics is rarely used to analyze mechanical systems, despite its universal applicability.

This unit focuses on *energy*, defined as a conserved, substance-like quantity with the capability to produce change. Work is de-emphasized, and is more accurately called "working", indicating the nature of "work" as a process of transferring energy into or out of a system via external forces. The 1st Law of Thermodynamics is used as the primary means of analysis of mechanical systems because of its fundamental, universal nature.

All energy interactions can be characterized as energy transfer mechanisms or energy storage modes, depending on how the system is defined. Energy storage modes are internal and potential energies, designated as \tilde{E} with corresponding subscripts ($\tilde{E}_k + \tilde{E}_{el} + \tilde{E}_g + \tilde{E}_{diss} + \tilde{E}_{chem} = \tilde{E}$). Energy transfer mechanisms are working (W), heating (Q), and radiating (R). A thorough discussion of the concepts of energy storage and transfer, including a contrast between the traditional and Modeling perspectives can be found in the TeacherNotes in Unit 0: Energy-Preface. An excerpt¹ from this discussion dealing with representations of energy storage and transfer is provided in the Resources folder in this unit.

The relationship between energy storage and transfer is shown by the 1st Law of Thermodynamics, $\tilde{E} = W (+ Q + R)$. This is shown by the system schema below:



It shows that energy transferring into and out of the system affects the nature of the energy storage in the system. The 1st Law of Thermodynamics and the Law of Conservation of Energy state that the algebraic sum of these energy changes and transfers must add up to zero, accounting for all changes relative to the system. This crucial concept is incorporated into the pie chart and bar graph representational tools used in this unit.

The power of using the 1st Law of Thermodynamics for analysis is that it makes it possible to take into account the internal structure of the system, since energy dissipated by frictional forces can be accounted for as energy stored internally, E_{diss} .

¹ See RepresentEnergy.doc in the Resources folder.

In its expanded form, the 1st Law of Thermodynamics is $W+Q+R = \Delta E$,
where $\Delta E = \Delta E_k + \Delta E_g + \Delta E_{el} + \Delta E_{chem} + \Delta E_{diss}$

So for mechanics (in this unit), leaving out Q and R)

$$W = \Delta E_k + \Delta E_g + \Delta E_{el} + \Delta E_{chem} + \Delta E_{diss}$$

Notice that when the internal structure of the system *can* be ignored, the work-energy theorem appears naturally, from the 1st Law: $\Delta E_k = W$ assuming no other storage modes are involved. ($\Delta E_{diss} = 0$, if internal structure is ignored.) The work-energy theorem should no longer be used as a generic catch-all equation. It has limited usage because in many situations (frictional systems, and system deformation) its use is conceptually inaccurate since it ignores internal structure. The use of the 1st Law of Thermodynamics is much more comprehensive.²

The concept of "working" is not introduced first. Instead, the idea of energy storage is developed first, by revisiting the Modeling paradigm labs and using energy pie charts to account for all the energy storage modes involved in a given situation. (See the corresponding Unit Addenda for details on these analyses.)

Worksheet 1

This worksheet gives students practice in system definition and energy storage analysis, using pie charts as a qualitative means of analysis.

When discussing ws1, the analogy of money is helpful to clarify the ideas of energy transfer and storage, as opposed to different forms of energy. Money can be "stored" in many ways - a wallet, a checking account, an IRA, etc, but it is still all money. Only the way it's stored has changed.³

Hooke's Law Lab

Apparatus

Two springs (one of these could be the spring used in the energy transfer lab later in the unit)
Lab masses and meter stick, or
Force sensors

Pre-lab discussion

The Hooke's Law lab is used to introduce the quantification of elastic potential energy. Students know that as the force acting on an elastic system increases, so does the energy stored in the system. The purpose of this lab is to determine the relationship between the force and the amount of stretch of a spring.

Lab performance notes

Students be cautioned against over stretching the springs.

If they are hanging weights on the springs, they need to be reassured that for a stiff spring it is OK for the spring to not begin stretching until a threshold force is reached. When they graph their data, they should be advised to plot F vs x, despite the fact that force was the independent variable (there was a precedent for doing this in unit 3).

This problem is trivial if they use force sensors and use stretch as their independent variable. However, students need to be cautioned against entering 0,0 as a data point.

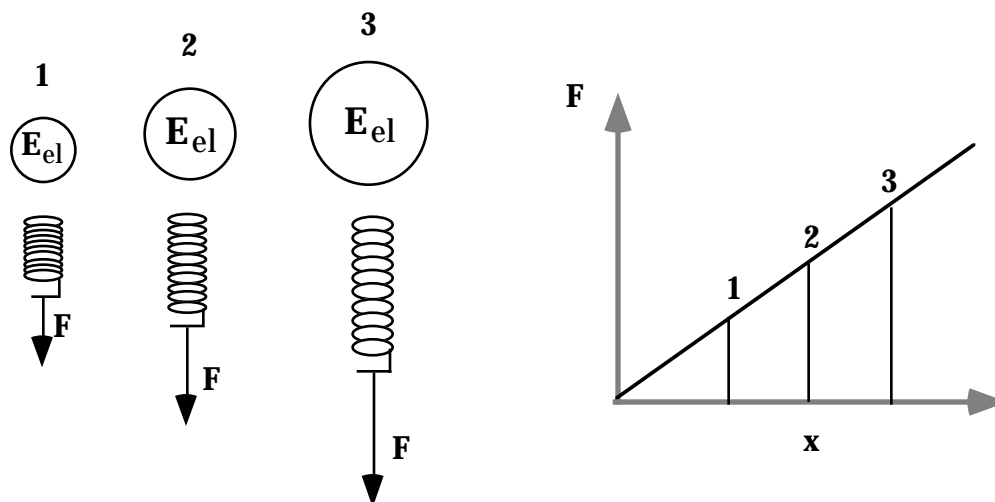
² For more details on these conceptual inaccuracies, see Unit 0 - Part 3: Justification and Goals Misconceptions

³ See Unit 0 - Part 2: Energy Transfer and Storage for more details on this analogy.

Post-lab discussion

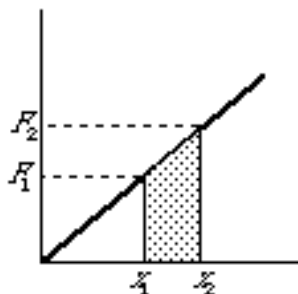
Students should find that force is proportional to the stretch. The general equation for the graphs should be $F = kx + F_0$, where the slope, k , indicates the force per unit length of stretch, and the intercept, F_0 , indicates how much force must be applied before the spring begins to stretch in a linear manner. *After* discussing the *meaning* of the slope and intercept, you can use the terms spring constant for k and loading force for F_0 .

Now that they have discovered Hooke's Law, the students are ready for a discussion of the *energy* situation of the spring. A pie chart analysis of the situation as the spring is stretched shows the pies getting larger with each stretch, indicating more and more energy being stored with each increase in applied force. Correlating these growing pies with the F-x graph, it is not difficult to make the connection between the area under the F-x graph and the size of the pies, and thus define the triangular area under the F-x graph as the elastic potential energy stored in the spring. Notice - *no mention of work as the area under the graph!!!* That will come later!



Derivation of Elastic Potential Energy Equation

- 1) We defined the area under the curve to be the energy stored in the spring, $E_{el} = \frac{1}{2}Fx$.



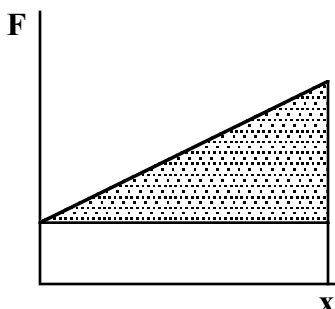
From the graph at left, one can see that to determine the work done when $x_1 > 0$, one can subtract the area of the smaller triangle from that of the larger.

$$E_{el} = \frac{1}{2}F_2x_2 - \frac{1}{2}F_1x_1$$

From the lab, you found that $F = kx$.

When this substitution is made into the equation above, one obtains

$$E_{el} = \frac{1}{2}(kx_2)x_2 - \frac{1}{2}(kx_1)x_1, \text{ which simplifies to } E_{el} = \frac{1}{2}k(x_2^2 - x_1^2).$$



- 2) What about the case of the stiff spring? While the area under the curve still represents the energy stored in the spring, only the triangular region represents energy that can be readily transferred to another storage mode. The graph can be modified by shifting the stretch axis upwards until it intersects the curve. You may decide to avoid this issue by choosing springs that exhibit a linear response to force from the outset.

Worksheet 2

At this point, ws2 can be used to solidify the understanding of elastic potential energy and Hooke's Law.

Quiz 1

Qualitative Description of Working as Means of Energy Transfer

Having established $E_{el} = 1/2kx^2$ as the quantitative measure of how much energy is stored in the spring, the next natural question is, "Where does this energy come from? The E_{el} pies get larger as the spring stretches - where did the increase in energy come from?

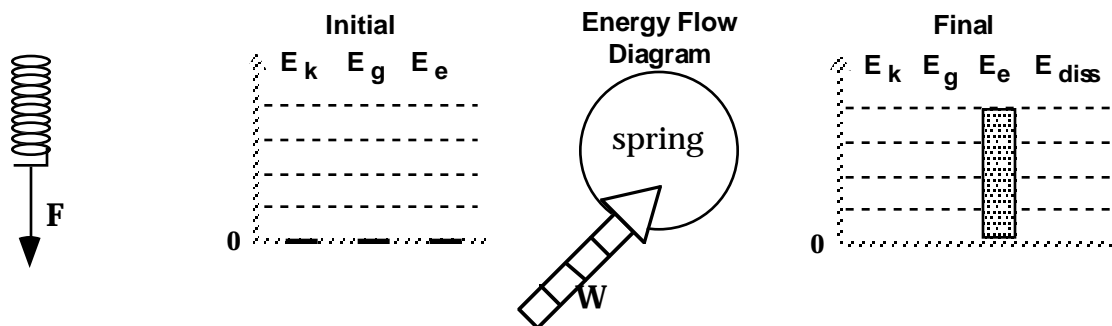
The spring is defined as the system. So the added energy is coming from a force *exerted by an external agent* (outside the system) that causes the spring to store energy. We define this transfer of energy into (or out of) the system via an external force as the process of working, W . An energy transfer such as this affects the energy storage in the system. It is helpful to revisit the money analogy here: depositing a check into the bank is a transfer that results in a corresponding increase in one's savings account (storage).

It is important to emphasize that the amount of energy transferred *equals* the change in energy stored. This will lead into the 1st Law of Thermodynamics and the Conservation of Energy.

So, the energy added by the external force on the spring is equal to the energy stored by the spring as a result of the process of working:

$$W = \Delta E_{el}$$

Use the bar graph schema at this point to incorporate the transfer representation.



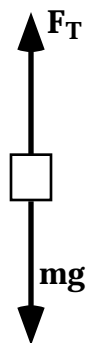
We have defined the area under the F - x graph to be equal to the energy stored. Since the energy stored is equal to the energy transferred by *working*, the area under the graph must also be equal to the energy transferred by *working*. So now we have two definitions of *working*:

1. Working: transferring energy into or out of a system by way of an external force
2. Working: the area under an F - x graph, where F is the force exerted by the external agent transferring energy.

Worksheet 3a

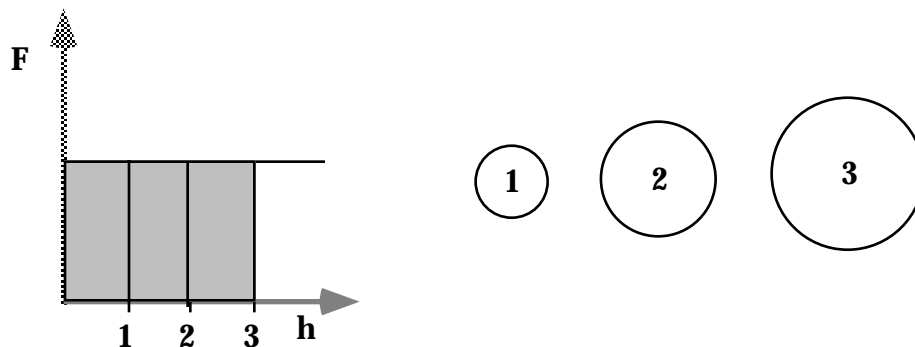
This worksheet introduces qualitative bar graph schema analysis involving working and various energy storage modes. It is crucial that the students define the system before doing the analysis of each situation.

Quantifying Gravitational Potential Energy



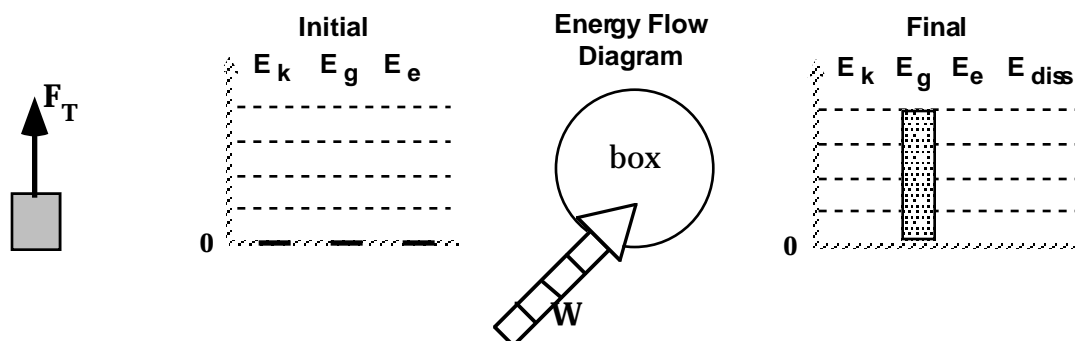
Let's apply these definitions of working to an analysis of gravitational potential energy, E_g . When an object is lifted near the surface of the earth, a force is applied. If the object is lifted at constant velocity, the lifting force, F_T , is equal to the weight, mg , the entire time.

When an F - x graph of this process is made, it can be seen that the force applied (F_T) is constant. As the object is lifted higher, the area under the graph increases, as does the E_g of the object, due to its position relative to the reference point where $h = 0$.



The area under the graph is rectangular, so $W = F_T \cdot h$. The lifting force is equal to the weight of the object, mg , since it's moving at constant velocity. So $W = F_T \cdot \Delta h = mg\Delta h$. Since the energy transferred, W , equals the resulting energy stored, E_g , $W = E_g$, so $E_g = mgh$, quantifying another energy storage mode.

Bar graph analysis is also important here: working is done, since the agent exerting the lifting force is *external* to the system. As a result of the process of working, energy is added to the system, and is stored as E_g . The work done (energy transferred) is equal to the energy stored (E_g), $W = E_g$.



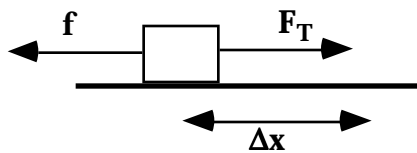
Calculating Energy Transfer due to Working

At this point, having firmly established the concepts of energy storage and transfer and the use of representational tools, we can use the standard textbook treatment of "work" to describe quantitatively how energy is transferred by working. Working is defined to be the transfer of energy by an external force acting parallel to the direction of motion, $W = F_{\parallel} \cdot \Delta x$.

Accounting for energy dissipated by friction

Where the standard textbook treatment of work runs into trouble is in accounting for the energy dissipated by friction. We suggest avoiding statements such as "the work done by friction" since the way we define our system (including the surface), friction is not an external force.

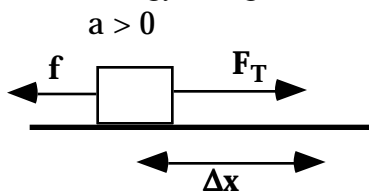
Working is done by an *external* force. If the force of friction is equal to the pushing force, all the energy transferred by working is dissipated by friction, and there is no other change in energies. Thus the kinetic energy remains constant, ($\Delta E_k = 0$) so the box is moving at constant velocity.



$$W = \Delta E \quad F_T \cdot \Delta x = \Delta E_{diss} + \Delta E_k;$$

since $\Delta E_k = 0$, $F_T \cdot \Delta x = \Delta E_{diss}$

If, however, the frictional force is *less than* the external applied force, then ΔE_{diss} is less than W , so some energy also goes to increasing E_k and the box accelerates:



$$F_T \Delta x = \Delta E_{diss} + \Delta E_k$$

Now is an appropriate time to discuss E_{diss} in greater detail. Where does dissipated energy go? The key is to recognize that where E_{diss} is involved, the internal structure of the system can no longer be ignored. Representing the system as a point-particle is inappropriate. This is especially important in the case of sliding friction in which the energy stored as E_{diss} results from an interaction between the object and the surface; both object and surface must be part of the system.

Dissipated energy is distributed among the kinetic and potential energies of the constituent particles that make up the objects in our system. The most obvious effect is that the temperature of the object(s) in the system increases. However, the potential energy of the individual particles (as determined by their arrangements) can also change. Since these energy storage modes alter the makeup of the system (as opposed to changing the state of the system at large) we assert that dissipated energy does not leave the system via macroscopic mechanical means. Instead, one must call on heating or radiating as processes that transfer energy from/to the system. A useful example is that of the disc brake rotors and pads. Students recognize that there is something different about the materials immediately after braking. After a while the temperature of these materials returns to normal, as energy is transferred to the surroundings by Q and R.

In any event, since we have defined the area under an F - x graph to be energy stored, the area under the F_k - x graph represents the energy dissipated by friction, so $\Delta E_{diss} = F_k \Delta x$.

So long as we restrict our discussion of energy transfer into or out of the system via working, then an operational rule could be stated: Dissipated energy remains in the system.

Conservative vs. Non-Conservative Forces

It is crucial to stress the effect that friction has on energy interactions in the analysis of working; it is important to distinguish between conservative forces and non-conservative forces. The standard treatment of work and energy tends to gloss over this potentially confusing issue. Friction is a non-conservative force because energy transfers are non reversible, whereas gravity is a conservative force

because the total change in energy depends only on the difference between the initial and final positions.

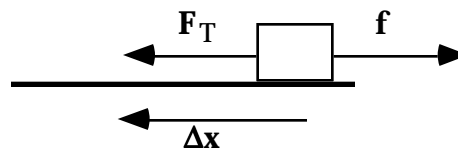
Let us first consider the case of lifting an object at constant velocity. [Assume the system is the object and the earth, so the agent providing the lifting force is external, and gravity is a source of internal energy]. A lifting force F_T is working on an object. As a result of the lifting work, the energy due to gravity increases: $W = \Delta E_g$. No other energies are involved. The amount of energy gained is only a function of the net change in position (displacement). No matter how many times one moves the object up or down, the final resulting increase (or decrease) in E_g depends only on the initial and final positions of the object, not the path followed.

One can better understand this statement by considering the signs of the lifting force and the displacement. The applied force is always upwards (+). If the object is moving upwards also ($+\Delta x$), the work will be positive ($+W$), indicating that energy has been *added* to the system, now stored as E_g . If, however, the object is lowered, F_T is still positive, but the displacement is negative, so $W = +F_T(-\Delta x)$. The negative sign on W indicates that energy has been transferred from the system, resulting in a reduction in E_g .

Unlike situations in which frictional forces are involved, these transfers of energy into or out of the system with change in direction are reversible. When the object is lowered, energy is removed from the system. But that "loss" can be completely recovered by moving back up to the original position. There is no energy "lost" to internal storage modes. The final amount of work done will be the sum of all these positive and negative energy changes resulting from the up and down motions. Ultimately the final change in E_g depends only on the object's initial and final positions.

This means that if the object is returned to its original position, there has been no net change in the system's energy, and thus no net working has occurred. Because the displacement is 0, the net energy transferred to the system as a result of working is also zero. Hence, we call gravity a conservative force.

By contrast, one *cannot* only consider the final displacement when the situation involves the non-conservative force of friction. Some energy is always going to be dissipated due to the physical interaction at the surface boundaries. The process of working by an external force results in some energy transfer that is NOT recoverable (via working).



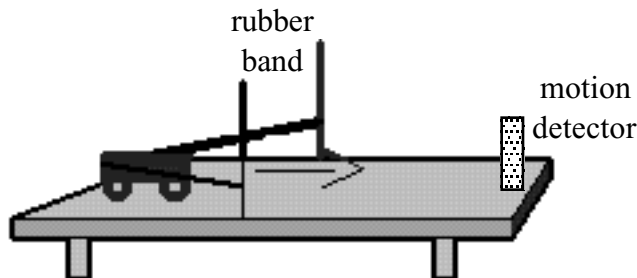
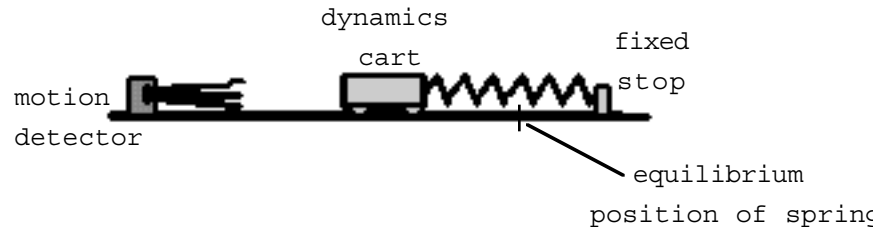
For example, consider the case where a box is pulled at constant speed through a displacement Δx , then is pushed back to the starting position. Since friction is involved, some energy is transferred to the system via working in *both* directions. Even though the displacement is 0, the system has more energy than when it started. In the case of a non-conservative force, the energy of the system always increases, regardless of the direction of motion.

Energy Transfer Lab

The quantitative relationships for E_g , E_{el} , E_{diss} , and W have been established. In order to solidify the relationship between energy storage and transfer and to quantify E_k , the Energy Transfer Lab can be conducted by one of three options. The first is shown here; the other two are in the Resources folder. The labs give physical, quantifiable experience relating energy transfer and the changes in energy storage.

Option 1. Elastic Potential Energy to Kinetic Energy Apparatus

springs (low k , with a known value - PASCO dynamics springs work well) or a rubber band
dynamics track and low friction cart
additional lab masses
motion detector
Graphical Analysis



Pre-lab discussion

- ☞ Define the system to be the cart and the spring.
- ☞ Remind students that as the spring is stretched it stores elastic potential energy.
- ☞ Note that the amount of energy stored in the spring decreases as the cart moves toward the end-stop and the final potential energy of the spring (rubber band) is zero. This means that as the spring pulls the cart, the spring loses potential energy. Ask the question, Where has this energy gone? Students will doubtless answer that the kinetic energy of the cart will increase. Some may even note that some energy will be dissipated due to friction.
- ☞ Ask what factors are likely to be involved in the kinetic energy of the cart; mass and velocity should be suggested by students.
- ☞ Lead them to recognize that while they will measure the *amount of stretch* of the spring, the variable to be graphed is the *energy* that was transferred from the spring to the cart, *within* the system.

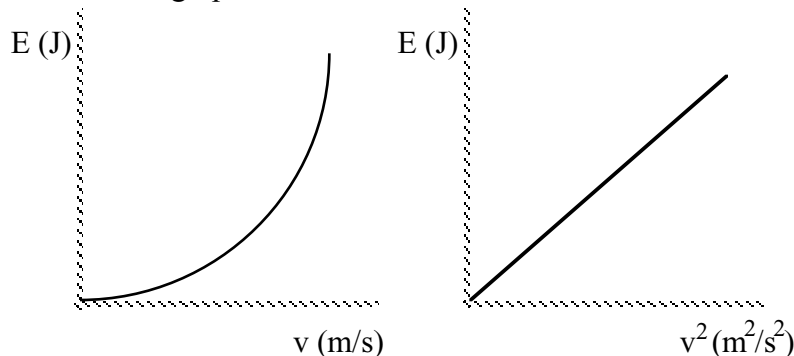
Lab performance notes

- ☞ If students have already determined k for the spring earlier in the unit, they can move right into determining the relationship between energy and velocity. A spreadsheet will simplify the calculations of the elastic energy and average velocity of the system. A sample can be found in the Resources folder.

- ☞ Be sure to warn the students not to overstretch the springs.
- ☞ The experiment can be expedited if different groups use different masses and then compare their results at the end of the lab.
- ☞ Students should perform multiple trials and use the software (LoggerPro, Motion, Data Studio) help them determine the maximum velocity of the system for each trial.

Post-lab discussion

Students should obtain graphs similar to the ones below.



They might not immediately recognize that the units of slope reduce to kg. Once they realize this, remind them that the slope is usually related in some way to a variable held constant during the performance of the lab. Then, have them post their slopes and system masses. If they conducted their experiments carefully, they should recognize that the slope is roughly half of the system mass. This suggests that the expression for the kinetic energy of the system is $E_k = \frac{1}{2}mv^2$. Ask students why the slope is only approximately half of the mass; induce them to see that some energy is dissipated during the transfer. Next, ask if this is a random or directed error; they should be able to account for why their slopes are generally too small.

After the lab, the use of the 1st Law of Thermodynamics should be emphasized as $W = \Delta E$, then addressing that $\Delta E = \Delta E_g + \Delta E_k + \Delta E_{el} + \Delta E_{chem} + \Delta E_{diss}$ so that

$$W = \Delta E_g + \Delta E_k + \Delta E_{el} + \Delta E_{chem} + \Delta E_{diss}$$

Then, to analyze a particular situation, one determines the changes in various storage modes as a result of an energy transfer by working, if any, and accounting for any increases in energy of the system as positive (ie, stretching a spring = $+\Delta E_{el}$) and decreases as negative (ie, an object slowing down = $-\Delta E_k$). The Law of Conservation of Energy says the algebraic sum of the changes in internal energy (ΔE) must be equal to the energy transfers into or out of the system.

The first step in quantifying energy analysis and solving traditional work-energy problems can be done with the bar graphs in order to facilitate identifying the energy interactions before applying formulas and numbers. Worksheet 3b combines the bar graphs with quantified problem-solving to help make this transition from qualitative to quantitative analysis. Worksheet 4 then leaves the use of the bar graphs or pie charts to the students, giving them just the written aspect of the problems. Again, students should use the structure of the 1st Law to set up their problems.

Worksheet 3b

Quiz 2

Worksheet 4

Work-Energy

Notice there is no specific mention of the work-energy theorem. First of all, it is not necessary to isolate that approach, since it will occur naturally with the use of the 1st Law of Thermodynamics. Secondly, it is not always conceptually accurate in terms of the energy interactions. The work-energy theorem is only appropriate for situations that do not involve the consideration of the internal structure of the system (no friction involved). As an example, here is how a traditional problem could be set up and solved using the 1st Law:

A 70 kg baseball player running at 4 m/s slides into home plate. How far did he slide before stopping, if the coefficient of friction between his clothes and the earth is 0.7? (assuming he didn't run into the catcher or anything else)

Energy analysis: system = runner and earth

Initial E: E_k Final E: E_{diss} $W = 0$ (no external forces)

1st Law: $\Delta E = W$ $\Delta E = -\Delta E_k + \Delta E_{diss} = W = 0$ so $\Delta E_k = \Delta E_{diss}$

$\frac{1}{2}m\Delta v^2 = f\Delta x$ - $\frac{1}{2}(70\text{kg})(4\text{m/s})^2 = -(0.7)(700\text{N})(\Delta x)$ so $\Delta x = 1.14$ m

While it may seem like a matter of semantics whether one speaks of "work done by friction" or "energy dissipated by friction", we think the distinction is important.

"In some instances, a quantity that *looks like* an amount of work done (e.g, $f\Delta x$) but is not real work done by (or against) that force is shown by the COE [Conservation of Energy/1st Law of Thermodynamics] equation to be *numerically equal* to an amount of real work that was done by some other force (e.g, F) and was dissipated."⁴

Since we have defined working to be a transfer of energy by an *external* force, and since friction is *not* an external force, friction does not do "real" work as we have defined it.

⁴ A Arons, "Developing the Energy Concepts in Introductory Physics", The Physics Teacher, Oct 1989, p 513.