

Unit VIII: Central Force Particle Model

1. Uniform Circular Motion

Define the relationships between velocity and force

Define the relationships between velocity and radius

Define the relationships between velocity and mass

Derive the mathematical model describing the relationship between force, mass, radius and velocity

$$v^2 \propto \frac{F}{mr}$$

$$F \propto \frac{mv^2}{r}$$

by paying attention to proportionality constant in each relationship the proportionality constant must be equal to one; therefore,

$$F = \frac{mv^2}{r}$$

2. Distinguish between centripetal and centrifugal force.

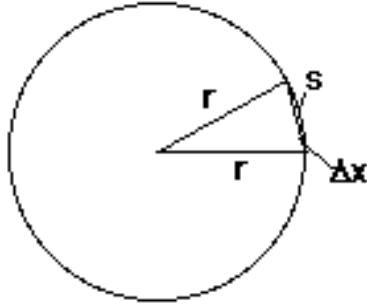
3. Force Diagrams

Construct force diagrams which display the force acting on an object undergoing uniform circular motion

Overview

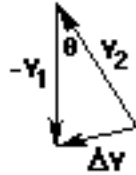
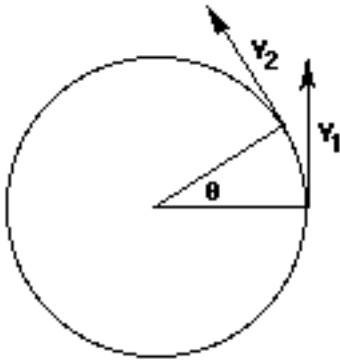
It is important for students to see that the relationship resulting from the lab is no different from the usual form of Newton's 2nd Law. Students also need to recognize that while the **speed** of an object undergoing uniform circular motion is constant, its **velocity** is constantly changing. The change in velocity, $\Delta \vec{v}$, is directed toward the center of the circle. This is quite different from the models developed earlier where the acceleration vector was either parallel to the object's velocity, or, in the case of projectile motion, always directed downward. The following derivation might be helpful.

1. Consider the circle of radius r below.



For a small θ , the arc length $s \approx$ the chord Δx .
The radii form the sides of an isosceles triangle with base Δx equal to the product of velocity and time. $\Delta x = \vec{v}t$

2. Now consider the circle below in which the velocity vectors for two positions are shown. Assume that the central angle θ is the same in both circles.



To the right of the circle is a diagram showing the difference of the two vectors as the sum of v_2 and $-v_1$. Because the stopper was traveling at constant speed, the magnitudes of the two velocity vectors are the same, making this second triangle also isosceles. The vertex angle θ is the same as the angle formed by the two radii in the previous diagram.

Because the triangles are similar, the bases are

$$\frac{\Delta \vec{v}}{\Delta x} = \frac{\vec{v}}{r} \quad \text{proportional to the sides. Thus,}$$

$$\frac{\Delta \vec{v}}{r \Delta t} = \frac{\vec{v}}{r}$$

substitute $v\Delta t$ for Δx

$$\frac{\Delta \vec{v}}{\Delta t} = \frac{v^2}{r}$$

rearrange terms

$$\frac{r}{a} = \frac{v^2}{r}$$

The vector sum $\Delta \vec{v}$, divided by the Δt , is the acceleration. Taken at the mid-time of the interval, it is directed toward the center of the circle.

$$F_{net} = m \frac{v^2}{r} \quad \text{from Newton's 2nd law.}$$

This force is commonly called the centripetal force because it acts towards the center of the circle. We advise that you make it clear that F_c is just shorthand for "net force acting towards the center of the circle" in much the same way that F_N is shorthand for "the support force acting perpendicular to the surface." Otherwise, students tend to think that F_c is a separate force; when they do, challenge them to identify the agent. **Worksheet 1** does a thorough job of reinforcing this point.

Instructional notes

Uniform circular motion lab

Apparatus

Centripetal force apparatus kit (see diagram below)

stopwatch

meter stick

balance

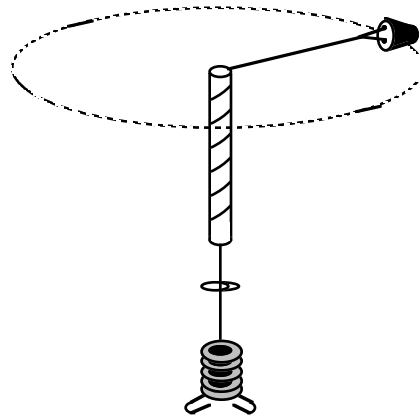
Graphical Analysis

suspended masses - increments should be in the 30-50 gram range

orbiting masses - (number 5 or 6 stoppers)

Pre-lab discussion

- ¥ Demonstrate apparatus at right to students. Elicit the variables that might influence the operation of the apparatus. Focus on the variables of mass, radius, velocity, and force of tension due to weight hanging from the bottom of the string.
- ¥ Discuss the characteristics for the system to be in equilibrium (i.e., spin object at a constant velocity with washer weights hanging at a constant position)
- ¥ Ask students how to manipulate the other variables such that velocity is the dependent variable in all experiments.
- ¥ Since they will be whirling the rubber stopper in a circle at constant speed, it will be undergoing uniform circular motion. When an object travels at a constant speed, its instantaneous speed is equal to its average speed, and can be calculated as such. Since the speed is constant and the radius of the circle is constant, the time to complete each revolution should be constant. The **time** to complete a single revolution is called the **period** of revolution. The period can be determined by timing a large number of revolutions (say 10 or 20) and by dividing the time for that many revolutions by the number of revolutions. The average speed (and therefore the instantaneous speed) of the rubber stopper will be the distance traveled in one revolution divided by the time to complete one revolution. Therefore you can determine the **speed** of the rubber



stopper by dividing the **circumference** of the circle by the **period** of revolution. The symbol for the period is T.

The velocity of the stoppers can be determined using the following relationship.

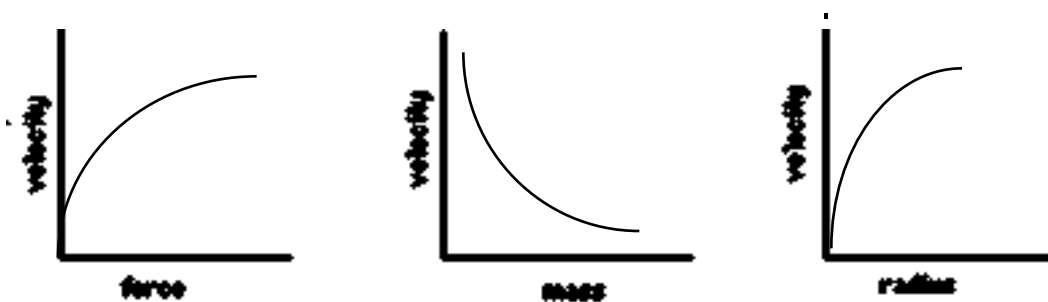
$$T = \frac{\text{time for } n \text{ revolutions}}{\text{number of revolutions}}$$
$$v = \frac{\Delta x}{\Delta t} \quad \rightarrow \quad v = \frac{\text{circumference}}{\text{period}} \quad \rightarrow \quad v = \frac{2 \cdot \pi \cdot r}{T}$$

Lab performance notes

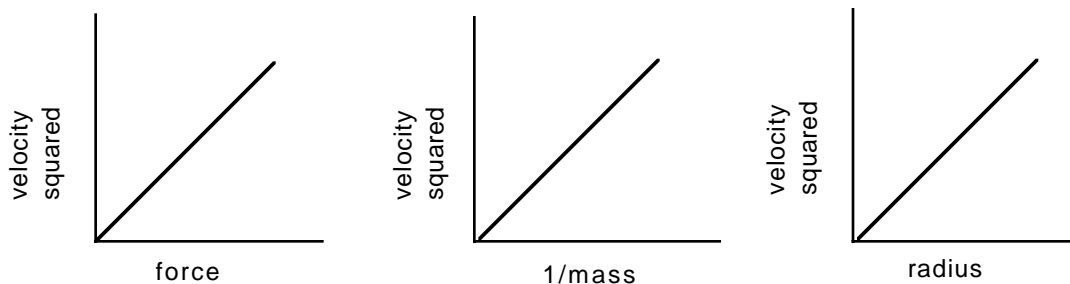
When determining the speed of the stopper, the student whirling the stopper(s) should have a lab partner measure the time required make a certain number of revolutions (at least 20). From this they can determine the time required for one revolution (period of revolution). They must also measure the radius of the circular path, which we will consider to be the distance from the glass tube to the center of the stopper. With the knowledge of the radius of the stopper's path and the time required to make one revolution, they should have no trouble determining the speed of the stopper.

Make sure that they understand the need to keep the other two independent variables constant while they are investigating the effect of one of the variables (force, mass and radius) on velocity.

The graphs they should obtain from their data should look like the ones below:



After linearizing the first graph, suggest to students that squaring the velocity should be the first step in the analysis of the remaining graphs. The final relationships obtained should look similar to the ones below:



Post-lab discussion

- ¥ Remind students that we've frequently found that the slope has a physical meaning -- it reveals something about the system.
- ¥ Have the students compare the units of the slope from the equation of each line to the units of the variables held constant. This should allow them to deduce that the variables are implicit in each relationship and are dependent on each other. The equations obtained for each part of the lab should look as follows:

$$v^2 = (\text{slope})F, \quad v^2 = (\text{slope})r, \quad v^2 = (\text{slope})\frac{1}{m}$$

$$v^2 = \left(\frac{r}{m}\right)F, \quad v^2 = \left(\frac{F}{m}\right)r, \quad v^2 = (Fr)\frac{1}{m}$$

- All of these have the same form, which can be rearranged to $F = m\frac{v^2}{r}$
At this point, you should show this same relationship is a result of the derivation of the acceleration of an object undergoing uniform circular motion (see overview).

Uniform circular motion lab simulation

The whirling stopper lab described above must be done with care in order to yield results that suggest the relationships that will eventually lead to $F = m\frac{v^2}{r}$. If you are unsure that your students are up to the task, or if you wish to save some time, the following alternative is suggested.

Apparatus

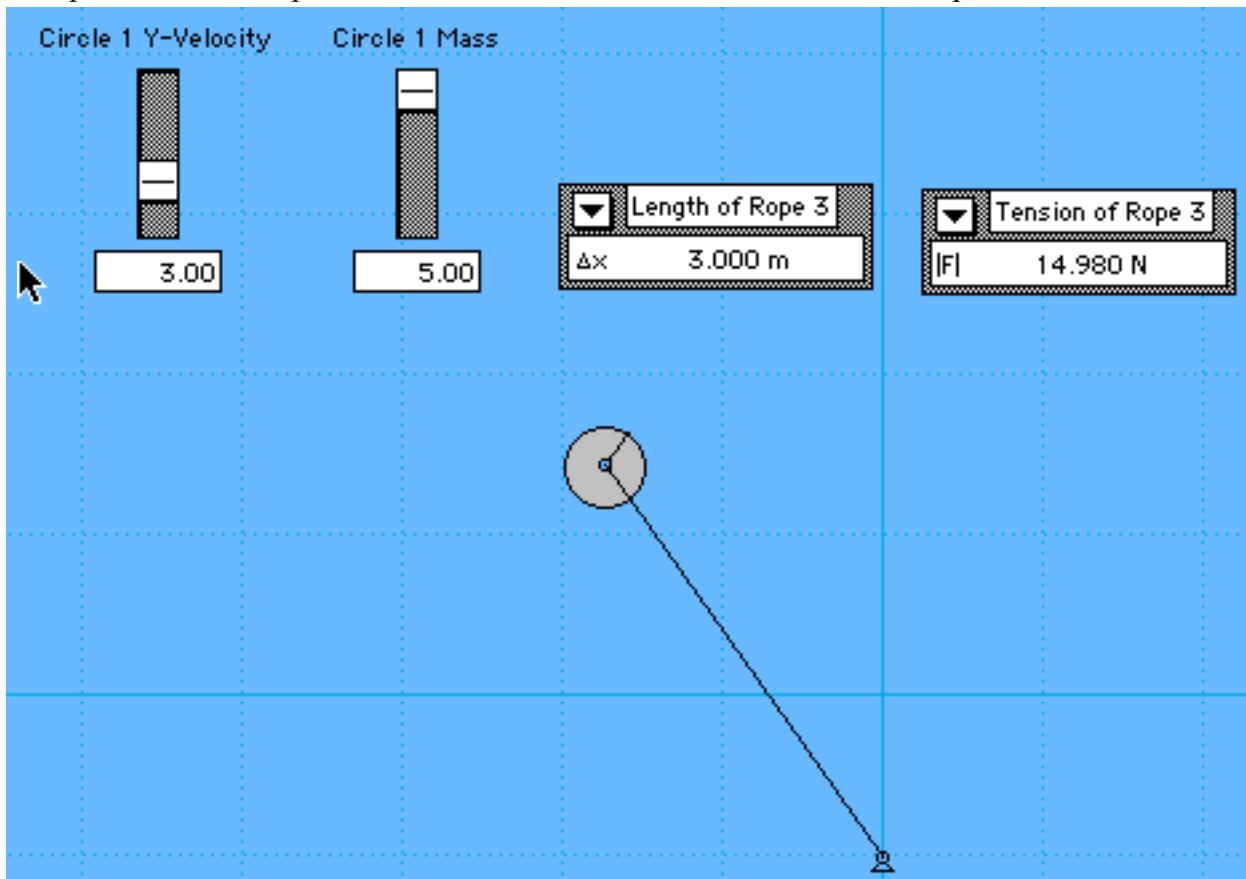
Interactive Physics
Graphical Analysis

Pre-lab discussion

Demonstrate the apparatus from the UCM lab described above. Call to the students' attention that the speed of the stopper is constant, yet since the direction is constantly changing, the stopper must be accelerating. Therefore, there must be net force acting on the stopper. Ask what could be providing this force. Guide their thinking to the tension produced by the hanging weights. Ask what are the variables that would affect the force required to bend the stopper into a circular path. Students should suggest mass, velocity and the length of the string (radius of orbit). Indicate that they will use a microworld created in Interactive Physics to investigate these relationships.

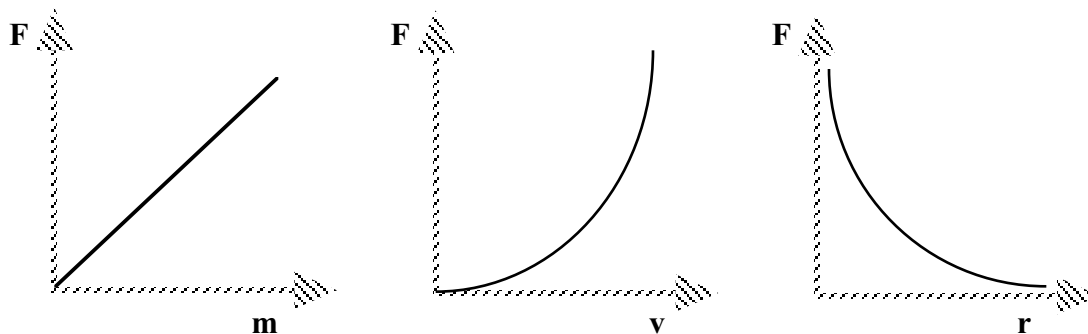
Lab performance notes

Depending on students' familiarity with Interactive Physics, you could have them build the microworld to perform the experiment, or set it up for them. The diagram below shows the required features to perform a simulated lab to determine the relationships between the variables.



In the World Menu, gravity must be set to [None] for this simulation to work. Sliders allow students to control mass and velocity. Varying the length is bit trickier. After performing the experiment with the 5.0 m rope, double-click on the rope and change its length to 4.0 m in the Properties box. This is preferable to starting with a short rope and increasing its length, since the rope will be slack. Even when you stretch out the rope, the values for length are not always well-behaved.

Students should record the values of the tension they obtain from the simulation, then use Graphical Analysis to produce graphs of F vs m , F vs v and F vs r . Their graphs should look like the ones below:



When students linearize the graphs, they will be able to write equations like the following:

$$F = km, \quad F = kv^2, \quad \text{and } F = k \cdot \frac{1}{r}$$

An examination of the units of the slope in each case will show that they are related to the variables held constant. So, (starting with the 2nd equation) units of kg/m suggest that the slope is the same as the mass divided by the radius. Units of kg m²/m in the 3rd equation suggest that the slope is the same as the square of the velocity divided by the radius. Finally, units of m/s² in the first equation can be obtained by dividing v² by r. It is a simple exercise to pick values of the parameters for a trial and see if they yield the slope of the graph from Graphical Analysis.

Students readily see that the three equations can be combined into $F = k \cdot m \frac{v^2}{r}$ and that $k = 1.0$.

Use of this simulation has the advantages of saving time and allowing students to do simpler calculations to obtain the key equation for this unit. The curves are perfect (naturally, since the "data" are calculated) and the students readily see that the slope is a function of the variables that were held constant.

Worksheet 1

Students examine how the normal force acting on a car varies as it crests a hill and bottoms out in a depression.

Quiz 1

Worksheet 2

If you do this unit before energy, then skip Q 3.

Model Deployment - Law of Universal Gravitation

This would be an appropriate place (if you so choose) to introduce the Law of Universal

Gravitation and then set it equal to the centripetal force $G \frac{Mm}{r^2} = m \frac{v^2}{r}$.

Worksheet 3

Worksheet 4

Unit Test