Putting More Models in Modeling
(Matt Greenwolfe)

The Greenwolfe folders within each unit contain a compilation of materials that reflect how I use the Modeling Approach to Physics Instruction. Only some of the material is original with me. I have borrowed extensively from others who have graciously shared their materials with me. I offer the same invitation to borrow from what I provide. For example, this paragraph is copied almost verbatim from Mark Schober, a physics teacher from St. Louis.

There's another reason I would like you to borrow from these materials. Right now, they work for my teaching style with my students at a certain school. I think they contain important ideas that need to become part of the standard modeling materials, but others need to use them, tweak them, write other problems to fit with the approach, and innovate so that we end up with something more generally useful. So please adapt these materials to your situation, and share what you've done.

General Philosophy:

In several lengthy conversations with David Hestenes, he emphasized to me the central importance of models to modeling instruction. Eric Brewe crystalized this in his recent paper in AJP where he contrasts "focusing the curriculum on 6 – 8 general models" with "14 or so textbook chapters" and "hundreds of important principles." Students see 6-8 models "as a manageable body of knowledge." Further, "This organization matches expert organization in which a few fundamental principles are viewed as requisite for a very broad understanding." [Eric Brewe, *Modeling theory applied: Modeling Instruction in introductory physics*, Am. J. Phys. 76 12, December 2008]
What gets students to organize their thinking around models? According to David, it is when they refer to the model by name while trying to explain their own reasoning. In that moment, the model starts to become a single entity that can be viewed in relation to other models. The student has zoomed out to a powerful conceptual level. I had to ask myself how often that occurred in my class.

In Dwayne Desbain's doctoral dissertation, he brought up the issue of models with students who had completed a high school modeling physics class and found "students left modeling high school classes without seeing how models were useful. They were just something the teacher made them do." He made similar observations about the use of diagrams and graphs. Dwayne's conclusion was that asking students to identify the models they were using without a task that *authentically required* models would leave students unconvinced of their value. To remedy this, he wrote open-ended problems that describe a physical situation, provide some quantitative information, but ask no specific question. Models then became authentically "necessary to figure what its possible to know" and diagrams and graphs became " necessary to link equations to models and to organize thinking."

Such problems are already a small part of modeling courses. All lab practica, for example, have some of these characteristics. Colleen Megowan studied the productivity of discourse in modeling courses and called such problems as lab practica or Dwayne's open-ended problems "practicing-with-the-model" problems. There are many variations, but all practicing-with-the model problems have in common that students cannot solve them without thinking in models and using diagrams and graphs. Practicing-with-the-model problems open like a funnel, rather than narrow to an insignificant point we refer to as "the answer." Exclusive use of equations and plug and chug utterly fails. Colleen found that the single most significant factor in fostering productive discourse was the problem type. Practicing-with-the-model problems generated the best discussions.

My high school students found Dwayne's problems to be overwhelming. As I struggled to help them meet the challenge, I wondered what help I could give them when the benefits of the task depended precisely on its open-ended nature. Students were supposed to explore the situation by applying models, not follow a set of steps I provided for them. The materials I provide are focused around four aids that successfully resolve this paradox.

Four Aids to Solving Practicing-with-the-Model Problems

1. *Matched Modeling Problems* – My students were overwhelmed with Dwayne's problems in part because the practicing-with-the-model problems lacked authenticity when you only have one model. So they didn't really take it seriously or practice the necessary skills until about unit V, when we had four models and all of a sudden things were very complex. By providing a matching set of problems with identical quantitative information but different physical situations, matched modeling problems make the students use something other than the given information to decide whether their model applies. You can see examples of these in units II and III. They generate rich discussions in class about the criteria to use to decide whether a model applies or not, and start preparing the students early in the year for more complex tasks.

2. *Scaffolding* – I wrote a scaffold of problem-solving steps to guide students through the open-ended problems. However, it differs greatly from the standard problem-solving procedures promoted in almost every physics textbook. You can see the scaffold in detail in the example problems, but briefly:

(a) Identify which model or models applies to the problem and explain your choice.
(b) Draw at least three (later in the year four) diagrams and/or graphs to describe the situation. Choose the ones you find most useful.
(c) Determine what it is possible to solve for and clearly show your method of solution.
(d) Show how to derive one of the equations you used from a graph. Do not repeat an equation you showed how to derive in a previous problem.

Initially, I thought that by gradually removing the scaffolding, students would eventually come to understand what it meant to "model this situation completely" and would no longer need the steps as a reminder. Surprisingly, this was not successful. Instead, I found that I had to alternately provide the scaffold and then suddenly yank it away. There had to be a very short time between solving a problem with the scaffold and encountering a similar problem without it so that the students got the point. You'll see I do this at least once and sometimes twice in each unit.

3. *Model Summary Boards* – In addition to frequently using the names of the models while engaged in zoomed-out reasoning, students must also know the content of the model. Once each model is refined enough, which occurs at different points during the deployment phase with each model, I have the students make a model summary board. The rules are these:

- Your summary must contain everything necessary to solve any problem to which this model applies.
- Your summary must fit on one whiteboard, be uncluttered, and use large writing so that it can be seen across the room.
- Your summary must use multiple representations: diagrams, graphs, equations, words, proportionalities.
- Your summary must pay particular attention to how to derive equations from the graphs.

This is not the typical cheat sheet sometimes allowed on college physics exams, on which students cram countless example problems in hopes the test problem will be similar. This task forces the students to decide what is most important and must be included. Aside from the four rules, the students must be free to decide how to present the information, what to include and what to leave out. When sharing the boards, different groups take different approaches and it is always interesting to compare and contrast.

When I have them, I have included pictures of students model summary boards for you to get an idea of what they look like.

4. *Concept maps* – To solve a practicing-with-the model problem, a student must first take the big picture, think of each model as a separate entity, forget about details for the time being and figure out which models apply to the problem at hand. Then the student has to zoom in a little and use diagrams and graphs from the model or models they chose to organize the given information and their thoughts and figure out which equations apply. Finally, they must zoom in to the details and carry out a procedure to solve for answers. But there is also a bigger picture, and that's the way the different models relate to each other and fit together to build Newtonian theory. The purpose of the concept map is to get students to think at this level. Creating a concept map is another opportunity for the students to use the names of the models as they explain their thinking.

I first ask students to make a concept map during unit V, when we have four models. I could start one model earlier, but an awareness of the structure seems to start to build after developing the four models. I pull out a blank whiteboard, ask the students to list the models we've studied, and each time they mention one I write its abbreviation (CVPM for constant velocity particle model, for example) on the board and put it in a bubble. Once we have all four, I point out that, "I placed them at random places and at random angles to each other, but perhaps it shouldn't be that way. Perhaps certain models should be closer together and others farther. Perhaps one should be inside another, or partially overlapping, or connected, or color-coded the same color. Use your imagination to express yourself, but in some way show how all the models relate to each other without putting any other words on your board other than the abbreviations for the names of the models."

Once again, students take a variety of creative approaches and usually come up with several different ways to express the same underlying structure. Some are more sophisticated than others. The least sophisticated just stack them in the order we studied them. The most sophisticated contain a great deal of information with very little ink. As the group presents, you begin to see meaning in even the smallest details of how they arranged things. I have included pictures of students model summary boards at various points in the course so you can see what they look like.

After students have shared their model summary boards, I share with them the following metaphor. "Some students organize their physics in their minds by putting all their equations and facts on a single mental page. I can tell those students because when I ask them a question, it can take them a long time to respond. Meanwhile they're looking at me like I just asked them to recall the most insignificant detail. Its taking time because they are on that mental page and have to sort through all of physics, or all that they've studied so far, to find what I've asked for. They're smart, the information is there, but they haven't organized it. By contrast, you can organize your mental knowledge of physics like a web site. Your home page is your concept map. Each model abbreviation on your concept map is a link. When you click on a link, you get the model summary board for that model – not every single possible detail, just the model summary board. That's all the important stuff that you really need. When your thinking is organized like this, you can quickly recall the information. This is also how you should solve problems - models to model summaries to the fine details."

How I use the worksheets

Another observation from Colleen Megowan's dissertation was that discussion is more productive when students all encounter the problem for the first time in class, all groups solve the same problem, and the class discussion is in the form of a board meeting (in a circle) rather than a formal presentation with the presenting group at the front of the room. I took her observations to heart and divided the worksheet problems in half. One-half are solved in class as whiteboard problems as described above and the other half are given as homework and graded, but never discussed in class. In the documents provided, the ones with "wb" in the title are the whiteboard problems for class, and the ones with "ws" in the title are homework problems, usually easier but sufficiently different from the whiteboard problems that students must get the concepts from class to apply to the homework.

This system may not work in your situation, so feel free to take the problems and use them as you see fit. I would recommend, however, that at least the students first exposure to practice-the-model problems be in class as described above and not for homework. This way you can negotiate more smoothly with them what it means to solve such a problem. Whenever the complexity ramps up, you might consider working a problem solely in class again.

Other files – In the folders with each model, you will find practicing-with-the-model problems, and photographs of whtieboards. You will also find variations on paradigm labs, suggestions for lab practica, and ways to make the modeling curriculum more graphical. This involves some specific graphical problems, often borrowed from Mark Schober, but especially includes a graphical approach to forces and momentum. In particular, I believe that modeling needs to take a more graphical approach to forces so that solving force problems remains a zoomed-out, conceptual exercise and does not become the rote application of a procedure. Participants in my summer 2008 modeling workshop developed the new graphical representation for momentum. When I used it with my students, there was no question it was better. Look at the materials and see what you think.

In conclusion, I have developed an approach and provided materials designed to put more models in modeling instruction, and to foster the more frequent use of graphical methods for solving problems. I believe that they successfully do so in my classroom, but in the tradition of physics education research, we need some comparison from other classrooms and other teachers and we need to collect some objective data not only to verify the hypotheses, but also to find the weaknesses and continue improving.

Update July 2010: For many years, I have been an advocate of a graphical approach to problem-solving in kinematics, gradually evolving to the point that I didn't even derive the kinematics equations, but just taught students to use area and slope of the velocity graph to solve. Sticking the matched-modeling problems into the mix distracted me away from that approach by asking students to formally derive equations. This was much less successful, so I've revised CVPM, CAPM and 2-D motion to be compatible with a graphs first approach, and in fact close to a graphs only approach. They don't request derivations of equations at all. I delay requests for symbols-only derivations of equations until later in the course when students are more familiar with these models and need to use them in coordination with other models to solve longer problems. At that point, I want to keep reminding some students where those equations they memorized came from, and I want to push other students to now develop symbols-only equations from their graphical understanding. None of this is necessary earlier in the course.

In rereading Arons for the summer workshops, his recommendation that students must add change in velocity to velocity step-by-step with simple numbers reminded me of Mark Schober's post-lab problems constructing graphs from a table of calculations. I have modified those problems to use graphs instead of a huge table of prescribed calculations that I felt would get reduced to a rote process by too many of my students.

Finally, in reviewing Bob Biechner's TUG-K, I realized the importance of his graphical problems with y-intercepts and curves, as they really tested the idea of slope and needing two points to compute slope. I've added problems addressing this as well.

There are two other changes specific to CAPM. I've observed the following stages of development of the acceleration concept among my students.

Sequence of refinement of concept of acceleration:

1) accel = velocity (motion not discriminated, all just motion)

2) accel = speeding up
3) accel = how much speeded up *in one unit of time*
   a) 2 and 3 include developing the concept of instantaneous velocity.
   b) important misconception: accel = velocity/time
4) Accel = how much speeded up (positive) or slowed down (negative) in one unit of time
5) accel = change in velocity in one unit of time (includes negative direction).

Most of my students arrive somewhere around stage 2, get to stage 3 after the paradigm lab and advance to stage 4. Not enough get to stage 5 during the unit, and therefore continue to struggle with interpretation of graphs when motion is in the negative direction, or changes direction.

I've redesigned this unit to allow students to land at each phase and get comfortable and then refine their concept. I take the opportunity to talk to them about refining an idea so they get the sense of retaining and including their previous ideas while improving them. This is important so they don't feel like they are always wrong.

One big aid to this was to bring back the original format of modeling worksheet 4 for this unit, with it's large table, confined space for each problem and suggestion to draw a velocity-time graph. I felt this taught the central importance of the v-t graph in particular, and also emphasized area and slope in a way that the “nicer” reformating or splitting up of these problems did not accomplish as well. These problems all involve speeding up or slowing down in the positive direction, so serve as a good emphasis of stage 4 of the concept development.

I wait until after this worksheet to introduce movement in the negative direction or changing directions.

Unit 1 now has a new emphasis on scaling and proportionality targeted at my non-honors juniors and seniors. Everyone is required to take physics at my school, so this group includes our lowest-performing math students. The following post to the list serve summarizes best what I'm trying to accomplish here.

Date: Thu, 9 Sep 2010 10:22:55 -0400

From: Matt Greenwolfe <matt_greenwolfe@CARYACADEMY.ORG>

Subject: linearizing

To: <MODELING@ASU.EDU>

Just want to second all of Joseph's comments. The reason I don't do linearizing is the mathematical level of the students I'm teaching in the non-honors sections of physics. Instead, as Joseph suggested I do a lot of work with them on proportional reasoning and scaling in unit 1.

For example, I introduce the squared function by having them measure the area of a parabola (cut off with a line at the focus) by box counting. Then I ask what would happen to the area if the base and arc length were doubled, leading them to invent the scaling rules.

We graph area vs. base length for all the parabolas and get the squared graph and function. Although we aren't linearizing, the fact that it turns out to be a squared function is no surprise because of the previous work. We focus on the meaning of the slope as giving the percentage of a "square base" that the parabola covers. We follow up with some regular geometrical figures and "amoebas" and discuss why the scaling arguments work for all of them.

Then I give them some questions about scaling in other geometrical situations, many of them drawn from Arons. Then we do an experiment where they encounter the squared proportionality in a cause-effect context more like we will use it the rest of the year. I don't cover the direct and inverse functions quite so thoroughly, but still discuss and use the scaling/proportional reasoning aspects as well as the equation and graph shape.

This all takes a lot of time, but as I've refined and improved it over the past few years, it keeps making a bigger and bigger difference in students later understanding. I think I get at least some of this time back.

At the end of the day, if the students can associate a graph shape with the equation, find the value and units of the constant in the equation and interpret its meaning when appropriate, and apply scaling/proportional reasoning when it will get them answers quicker and easier than using the equation, then I am satisfied. For the lower-level math students, the process of linearizing was mathematically complex enough that it was hindering their ability to comprehend the proportional reasoning. Doing things this way, they are really impressed with how proportional reasoning can save them a ton of algebra, and this has quite the opposite effect. It simplifies things, so they keep using it and really learn it.

I am sensitive to the argument that linearizing by hand can help prevent the computer curve fitting from becoming a mysterious black box that just produces a function. But at a certain math level, it does not serve that function because it increases the mathematical complexity too much. So I'm making what I think is the best compromise for the students I have.

Matt