Action Research Project:
A Study of Student Comprehension and Manipulation of Vectors Based on a Vector Unit

Submitted to Arizona State University in summer 2006.
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Description

Vectors, while seemingly straightforward as a quantity with magnitude and direction, apparently are not as fully understood as we hope that they may be. Although an important and prevalent topic in both mathematics and physics (think of the numerous vector quantities: displacement, velocity, acceleration, force, fields, momentum, etc.) the topic of vectors are typically given only a cursory treatment in textbooks and instruction. In this study, we will attempt to develop introductory materials on vectors to increase students' comprehension, comparison, and manipulation of vectors as determined by student gains on a Vector Concept Quiz. Even though this unit is not intended to be an extensive and comprehensive study on vectors, we hope to expose students to a range of experiences, guiding their thinking and management of vectors to a free-vector model focusing on the effects of vectors, translating the mathematical concept into a tangible situation for the student (Poynter & Tall, 2005a).

Rationale

The typical high school physics curriculum deals briefly with the description and treatment of vectors despite its pervasiveness of vectors throughout the curriculum. This seemingly "obvious" concept is by no mean obvious for our students (Knight, 1995). Physics teachers often expect that students have had an introduction to the concept of vectors, at least graphically, in their math classes. Yet, in studies done at the introductory university level physics course, researchers have found that one-half to two-thirds of students did not have the necessary skills to address vector addition or subtraction relating to force and acceleration (Flores & Kanim, 2004; Knight, 1995).

With the numerous applications of vectors in a physics curriculum, it is curious to note that textbooks and instruction often provides for a brief handling of the subject. Instructors often assume that a reading and homework assignment should be adequate to develop a thorough understanding of vectors for application in future topics (i.e. acceleration and forces). However, students would probably benefit from a more thorough study of vectors and their properties (Arons, 1997).

Students often have difficulty distinguishing the types of vector quantities, and as a result have trouble carrying over skills regarding vector manipulation from one quantity to another (for example displacement vector addition versus velocity vector addition) (Arons, 1997). Having students focus on the effect of the vector's actions may provide some assistance for students trying to interpret differing situations that normally would prove difficult. (Poynter & Tall, 2005a). Shifting students' focus to the effect will hopefully help in their progression toward understanding and comprehending vectors.

Our unit is designed for use with the modeling approach to instruction. (Hestenes 1996) This method

a) helps students construct basic models around facts, representations, and common experiences
b) helps students describe, explain, or predict situations based upon their model

The models used in Modeling Instruction allow students to organize and explicitly formulate the relationships between and among things. The model we hope to develop among the students is that of a quantity with magnitude and direction that is capable of being able to freely manipulated via translation (with no effect on the magnitude or direction) and scalar multiplication (with no effect on the direction). This "free-vector" model (Poynter & Tall, 2005a) will hopefully carry over to the various vector quantities that the students will experience throughout the rest of the year.

Literature Review

Starting from students' math classes, students are exposed to vectors as quantities with magnitude and direction. This is usually dealt with using a graphical method using an arrow to represent the vector. As students enter their physics course, vector representations are now extended to include both geometric as well as algebraic approaches (Hecht, 1994; Hewitt, 1987; Murphy, Hollon, & Zitzewitz, 1986).

Vector comprehension and manipulation pervades and is used extensively throughout a high school physics curriculum starting from the beginning with velocity, acceleration and forces. Knight outlines the fact that such fundamental concepts such as "net force" and fields require a rudimentary knowledge and command of vector addition (Knight, 1995). Yet, in an investigation of student understanding of vector concepts at the university level found that ¼ of students who had completed a calculus-based physics course and ½ of students who completed an algebra-based physics course could not add vectors in two dimensions (Nguyen & Meltzer, 2003). Similar findings have been collected where one-half to two-thirds of students did not have the necessary skills to address vector addition or subtraction relating to force and acceleration (Flores & Kanim, 2004; Knight, 1995). Another study by Aguirre and Rankin investigated students' understanding of vectors for kinematics, with similar results (Aguirre & Rankin, 1989).

While it is clear that students typically do not bring a *working* knowledge of vectors to physics (Knight, 1995), students do bring an intuitive knowledge of the effects of, let's say multiple forces acting on a box (Nguyen & Meltzer, 2003; Poynter & Hall, 2005a). As a result, Poynter suggests that instructors have their students focus upon the effects of the action, rather than just the action itself. This focus upon the effect also frees the student to be more apt to adopt the "free vector" model for vector addition (Arons, 1997; Poynter & Hall, 2005a), overcoming the "attached to points" concept that most students hold (Arons, 1997; Nguyen & Meltzer, 2003).

This concept of a "free-vector" model is what Poynter has classified as the highest stage in a student's understanding of vectors with stage one being a one-dimensional journey or path notion to the free-vector in two dimensions (stage 4) (Poynter & Hall, 2005a, 2005b; Watson, Spyrou, & Tall, 2002). This conception of vectors as freely movable (assuming the direction stays the same) is further complicated by the absence of a grid (Nguyen & Meltzer, 2003), the true “free” vector.

Throughout the literature, a common agreement is that a good start for introducing vectors is with the displacement context (Arons, 1997; Knight, 1995; Nguyen & Meltzer, 2003) because they have had direct experiences with walking or traveling. Yet, care must be taken because while students have little difficulty in this introductory example, they have difficulty making the transition to other applications such as forces. One reason for this is that students adopt a “journey” metaphor which works well for the initial displacement example, but breaks down when working with forces (Megowan, 2005).

The solution does not lie with a simple demonstration, instruction, and a “this is what you do” approach, but first of all, more attention to the material of vectors (Arons, 1997; Knight, 1995; Nguyen & Meltzer, 2003). Considering the importance of vectors in physics, a unit on vectors seems like a clear progression.

Location

The unit was tested with students at two US public schools, Northside College Preparatory High School and William Tennent High School, and one International school in Lausanne, Switzerland. The Vector Unit and activities was adjusted according to the ability level of the students at each school. The unit was tested near the beginning of the course as background for the Mechanics Modeling Units.

Northside College Preparatory High School

Chicago Public Schools

This is a relatively new magnet high school (six years old) of around 1000 students. Physics first has been taught selectively at this school since its inception, but will be switching over to a complete physics first program starting this year so all incoming freshmen will begin with physics. There is only one level of freshmen physics with a maximum of 28 students per class. The school is on a block scheduling system where the teachers see the students twice a week for 100 minutes each class for the year. There will be three physics first classes that will participate in this study and four physics first classes that are taught using the modeling method, but with no vector unit used as a control group.

Willian Tennet High School

Centennial School District

William Tennent High School (WTHS) is a public school in Bucks County, a suburb of Philadelphia, PA that serves over 2000 students. Approximately 86% of the graduating class is moving on to higher education. WTHS is a Federally-funded Title I school. During the study, WTHS was on a 4x4 block schedule. Students attended 4 classes a day, each lasting 90 minutes, for 18 weeks. Under this schedule, students needed 4 science credits to graduate. Physics is an introductory, one credit course which meets every day for 18 weeks.

The vector unit will be tested during the first month of each semester. Six classes at WTHS participated in the research study: 4 Junior/Senior Academic Physics classes and 2 Junior/Senior Honors Physics classes. Class sizes ranged between 16 and 26 students.

International School of Lausanne, Switzerland
European Council of International Schools
This is a private school with a student population from around the globe. The school has the IGCSE and the International Baccalaureate Programme (IB) in place. The classes meet between 2-5 times a week, and for either 45min or 90min of instruction throughout the school year. The classes that will be participating in the study are 12th grade Higher Level IB, 9th grade IGCSE, and 8th grade General Science. There are between 6 and 22 students per class.

Methods

The questions that have prompted this study are best answered by a teaching experiment. From the literature review and our anecdotal experiences, one of the primary weaknesses in the classic instructional approach to vectors is the minimal exposure and practice of dealing with vectors for the students. Thus, we have attempted to develop brief "pre-unit" materials to address the primary areas of vector properties (magnitude, direction, and "movability") and the manipulation of vectors (addition, subtraction, and scalar multiplication). Although students may have received exposure to vectors in their middle and high school mathematics courses, it is clear from studies that there are not adequate connections made to their application with reference to physics concepts. While a thorough investigation of vectors could have been attempted, we felt that a "mini-unit" around a week long, would be sufficient for high school teachers to use in their classes to help their students develop a better understanding of vectors.

Unlike typical vector materials that are taught in pure math terms, we decided to provide the students with experiences or attach prior experiences to the material. For instance, rather than talking about a vector that is 3 units long and 90° from the horizontal, we can instead use a displacement experience of walking 3 blocks North, a situation that many students can relate to. In providing a sequence that is contextual, students not only will have an experience to relate to, but will also be able to directly apply the concepts that they are learning.

The unit we have designed is fairly brief. According to our estimation, the instructor should be able to complete the materials within a week. Ideally, the unit should be conducted prior to student use of vector concepts to obtain a good view of student preconceptions and pre-knowledge regarding vector properties and handling. The projected time for this unit would be before materials on constant velocity.

The instructional materials that were used to conduct this study consist of a concept quiz for use as a pre-test, post-test and an extended-post-test, a directional lab activity, a worksheet on drawing and interpreting one dimensional vectors, a two dimensional vector addition activity, a two dimensional vector addition worksheet, and extension worksheet, a free-vector investigation activity and accompanying worksheet. Whiteboarding occurred after student groups completed each of the activities, tutorials and problem sets. One-on-one interviews were conducted with selected students at the beginning of the study, near the middle, at the end of the unit.

Data was collected by anecdotal observations, recoding notes of student interviews along with collecting scores from pre-test, post-test and extended post-tests. on a modified Vector Concept Quiz (Nguyen and Meltzer, 2003) and student responses to worksheet problems.

The Study

Day 1: Administered Vector Concept Quiz (VCQ). Executed the map instruction paradigm lab and whiteboarded their process and representations of displacement. Introduced the graphical representation of a vector. Began Vector Worksheet 1 in class and finished for homework.

Day 2: Whiteboarded selected homework problems. Discussed the effects of a scalar multiple on vectors. Discussed vector addition. Preformed colored pencil activity to introduce the commutative property of vector addition and free vector properties. Began Worksheet 2 as extension of vector addition and subtraction. Completed worksheet for homework.

Day 3: Whiteboarded selected homework problems. Completed free-vector activity and whiteboarded findings. Began Worksheet 3 in class and finished for homework.

Day 4: Whiteboarded selected homework problems. Reviewed free-vector model.

Day 5: VCQ post-test.

Day 6: (After completion of mechanics unit) VCQ extended post-test.

While this unit was intended to be brief (week long), we allowed the timeframe and materials to be modified based upon perceived student understanding and level.

Analysis of Data

This study will produce pre, post and extended post VCQ scores, worksheets, and activity write-ups.

Student work was maintained in individual portfolios (i.e., worksheets, activity work, pre, post and extended post VCQ results). All work was identified by a pseudonym or student ID rather than by name in order to preserve students' anonymity during the analysis phase.

One member of the study did periodic interviews. In the course of these interviews the students reviewed their class work and, using stimulated recall and think aloud protocols, attempted to reconstruct their thought processes. Some interviews were audio taped and relevant footage was transcribed. Scratch paper used in the interview process was also preserved for later analysis. Interviews focused primarily on students' changing understanding and manipulation of vectors. We also looked for patterns in the VCQ pre-test that indicated particularly prevalent misconceptions, and attempted to explore these as well. In addition to a post test after the conclusion of the vector unit, we administered an extended post test at the conclusion of the mechanics unit to determine any retention of knowledge as well as to investigate how the application of vectors in the context of general mechanics concepts, will influence student adoption of vector properties (comprehension) and application (manipulation).

Using the student data (written work, VCQ results, and recorded interviews), we attempted to determine how students perceived and comprehended the concept and use of vectors and their subsequent manipulations via graphical and algebraic addition, subtraction, scalar multiplication, and free vector manipulation and trace how their thinking changed through the instruction. As further study, student written work could be used to determine a general group extent of understanding.

The VCQ (see Appendix B), which is a modified version of Nguyen and Meltzer's Vector Concept Quiz (Nguyen, Meltzer 2003) probes the basic properties and manipulation of vectors based on the literature review. A majority of the responses require the students to draw in responses or explain their reasoning allowing us to identify the misconceptions and student rationales.

Individual Field Tests Results

Field Test Results – Nathan Harada
Northside College Preparatory High School

Background/Population

Northside College Preparatory High School is a relatively new school (7 years old) and is part of the Chicago Public Schools. Northside is a selected enrollment school where the students are admitted based on test scores, as well as 7th grade performance and attendance. As a result, the school draws students from all around Chicago. Approximately 30% of students at Northside qualify for free or reduced lunch. There are around 1000 students at Northside with average class sizes around 26 students and science classes capped at 28 students. Classes are on a block scheduling system with 100-minute classes meeting twice a week.

Physics First has been in place at Northside since the school's inception, but it was initially for students entering with a minimum of Algebra credit. Two years ago, that math requirement was dropped for Physics First to allow any incoming freshmen to select the science sequence they wanted. This past year was the first year that all freshmen were required to enroll in Physics First and follow the Physics – Chemistry – Biology sequence. Students are required to take three years of science, with Biology being the only required science subject by the Chicago Public Schools.

The students who participated in this study were in three freshmen physics classes totaling 76 students. Of those, 11 students were not included in this analysis due to having incomplete test sets (see the Analysis section for more detail). Approximately half (32) of the freshmen in the study were enrolled in Algebra with the other half (31) enrolled in Geometry and a few students (2) were enrolled in Trigonometry. I also taught two sections of AP Physics C (Calculus based) with 36 students total in both of the classes. These students were either currently enrolled in a first year calculus course or was taking a second year calculus course. While these students did not receive the vector unit instructions, I have occasionally cited their pre-test scores as reference.

In the data (found in Appendix D), the class identification is as follows:

Block	Class Level
Block 1	Physics First
Block 2	AP Physics
Block 4	Physics First
Block 7	AP Physics
Block 8	Physics First

Curriculum/Methodology

This unit was part of the introductory unit that students went through at the beginning of the year to learn fundamental skills such as graphing, measurement, and linearization. This unit, as stated in our report, was intended to prepare the students for concepts involving vectors – such as acceleration, forces, and momentum – that would be covered later on in the course. This unit was started approximately three weeks into the start of the school year before we started the unit on constant velocity. While intended to be a brief one-week long unit, I took about one and a half weeks, including the post-testing.

The pre-test was given during the first week to all the classes (Physics First and AP Physics). This was intended to provide a baseline for the students as well as to maximize the time between the pre- and post-tests to reduce test familiarity effects. The extended post-test was administered during the last week of classes in June to assess long-term retention and understanding.

The curriculum that was followed was very similar to the one outlined earlier in this report. One significant change was made in the addition of an activity on multiple-force pulling between worksheets three and four. This was added to provide students with a real-time demonstration and interaction with vector addition and free vector manipulations. A write-up of this activity is included in our updated unit teacher notes.

In addition to the tests, student interviews were conducted after school during the week of the unit. Students were audio-taped and were asked questions including explaining their rationale on the pre-test, questions from the homework and in-class activities, as well as extension questions. Students would show their work and be asked to comment on, and clarify their thinking as it unfolded during classroom discussions and whiteboard sessions through a stimulated recall and think-aloud protocols. Due to time constraints, only a couple of these transcripts are included in this report.

Analysis of Data (Please Appendix D for the complete set of data)

The data consisted of the Vector Concept Quiz (VCQ) (see Appendix B) pre-, post-, and extended post-tests, student interviews, and anecdotal evidence. The VCQ was scored in two ways:

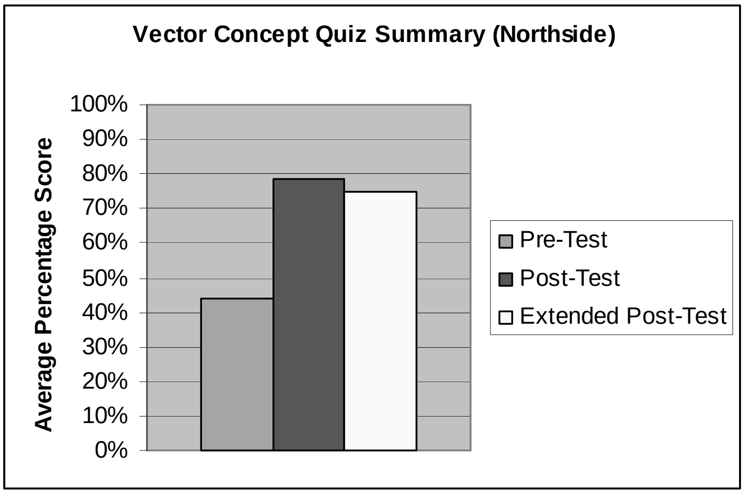
1. A numerical score was determined based on our scoring guideline found in Appendix C.
2. Student responses were coded based upon common student responses. The student response codes can be found in Appendix D.

Note that these analyses were completed only for students who had complete “sets” of tests. So if any of the test data for a student was missing, that student was removed from these analyses. As a result, the total sample size for this analysis was 66 students.

VCQ Numerical Analysis

This analysis was done for all the physics first classes combined.

	Average Percentage
Pre-Test	**44%**
Post-Test	**79%**
Extended Post-Test	75%
Pre-Post Normalized Gain	64%

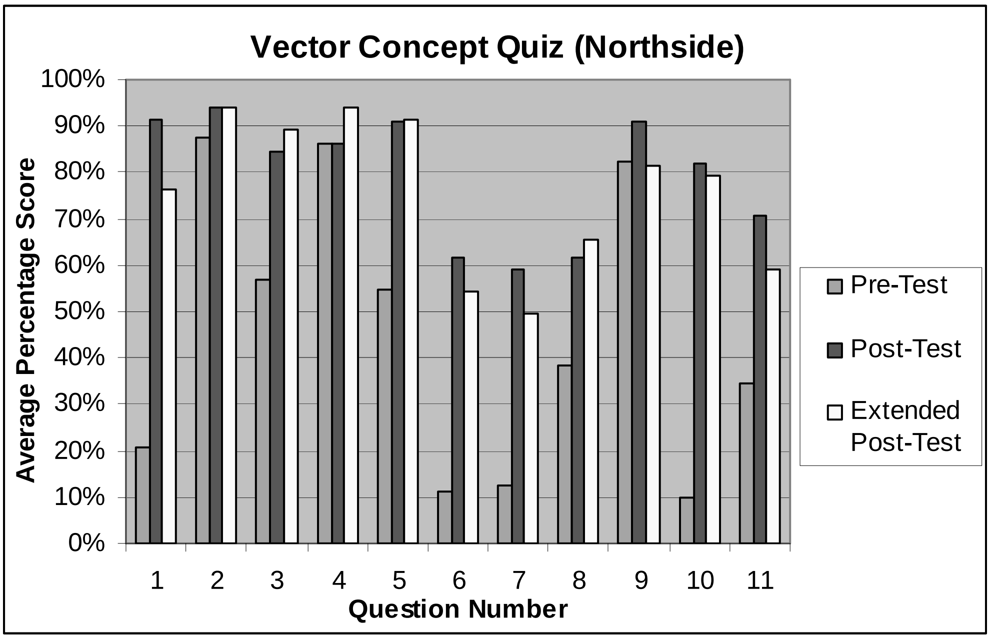


From this data, we can say that for this population, the vector unit was effective in improving student understanding of vectors based on the VCQ. It is also important to note that the students scored roughly the same on the extended post-test indicating student retention.

A brief analysis was also done for the question break-down for the numerical scores for the VCQ.

Average Score

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Pre-Test	**21%**	**88%**	**57%**	**86%**	**55%**	**11%**	**12%**	**38%**	**82%**	**10%**	**35%**
Post-Test	**92%**	**94%**	**85%**	**86%**	**91%**	**62%**	**59%**	**62%**	**91%**	**82%**	**71%**
Extended Post	**76%**	**94%**	**89%**	**94%**	**92%**	**54%**	**50%**	**65%**	**82%**	**79%**	**59%**



There are significant gains in almost all the questions except for 2, 4, and 9. A more detailed analysis was done using the coded student responses in the next section.

VCQ Coded Student Response Analysis

All the student responses were coded based on common student answers and rationale. To see the coding guideline and data, please see Appendix D. On some questions, percentage totals may add up to more than 100% because the student's responses often required several codes (for instance getting the correct magnitude, but not direction). While some of the coding guidelines are long, this would be an area of further study to group these responses based upon results obtained in this study. Interspersed throughout this analysis will be excerpts from student interviews, written student responses, and anecdotal evidence observed in the classroom.

Selected Results from Student Responses (to see the complete results, please see Appendix D)

1. *Write a brief description of what a vector is:*

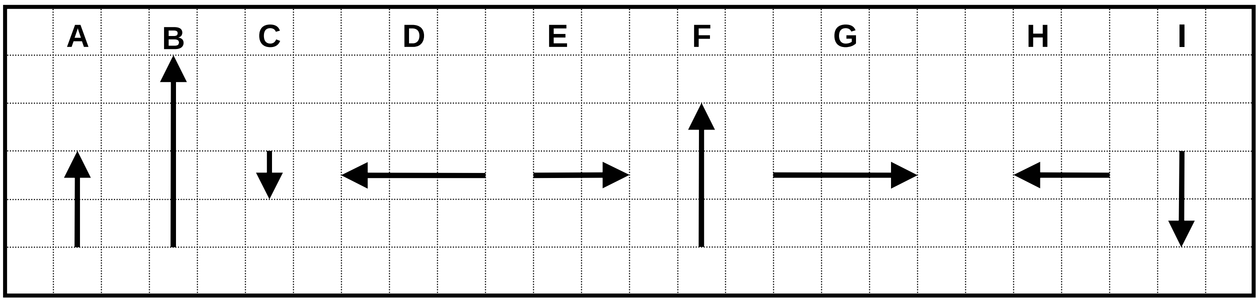
Student Response	**Pre Test**	**Post Test**
Direction	20%	97%
Magnitude or strength	9%	91%
Force (as an example)	23%	31%
Vector as a representation (line or arrow)	20%	43%
Explaining magnitude as “force of…”	5%	8%

A significant number of students expressed that a vector as the common representation of an arrow. Very few students seemed to make the connection that a vector described a quantity and could be represented by an arrow. This ties into the relatively high response describing a vector and showing direction. However, it must be noted that on the pre-test, only 58% of the students answered the question as opposed to 98% on the post-test. Through student interviews, I found that overwhelmingly, the students generally did not have any previous instruction or knowledge concerning “vectors.”

This view that a vector is a representation may stem from the prevalence of this representation in this assessment as well as the materials. Students were exposed to only a few examples of vector quantities such as displacement, force, and velocity. In almost all situations, students were asked to draw the “vectors,” possibly leading students to believe that the arrow was the vector.

One of the common associations with vectors for students is the “path” analogy for displacement and the “pulling” analogy for forces. We can see the second of these associations in both the use of force as an example (31% post) and as a description of the magnitude (8% post). However, unlike the literature on vectors, I did not find any student who used a path analogy when evaluating the vectors despite a prevalence of displacement vectors throughout the unit (paradigm lab and worksheets).

2. Consider the list below of pushes that you exert on a box and write down **<u>all</u>** pushes that have the same **<u>strength</u>** as each other. For instance, if push **W** and **X** had the same strength, and the pushes **Y**, **Z**, and **A** had the same strength as each other (but different from **W** and **X**) then you should write the following *W* = *X*, *Y* = *Z* = *A*.



Student Response	**Pre Test**	**Post Test**
All Correct	**82%**	**91%**
<u>Of the remaining:</u>		
Correct identification of 2 unit vectors	12%	8%
Correct identification of 3 unit vectors	11%	5%

Separation of horizontal and vertical	11%	8%

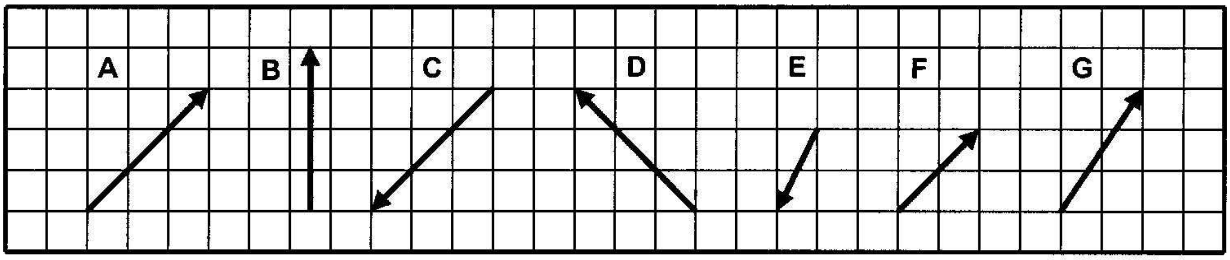
Most students on the pre- and post-test got this question correct. It is interesting to note that for about half of the students who did not get the answer of A=E=H=I and D=F=G, they got a partial answer that involved correctly matching up the 2-unit length and 3-unit length vectors, but only matched up the vectors that were either both horizontal or both vertical. Through student interviews, students expressed high confidence in explaining this question.

It is also interesting to note that in one of the student interviews, when asked about her answer of C=I, H=D, E=G, A=B=F, she expressed:

> "Last year with our science teacher, she showed us that if arrows pointing in the same direction would be equal to each other when we were doing the unit chapter. That's what she told us. That's what I remember, so that's what I put."

It is possible that this prior lesson in vectors overrode her intuition in evaluating this question. The last sentence probably is the key in that she incorrectly processed the information. We will see evidence of this again during question #8.

3. List all the velocity vectors that have the same **direction** as the first velocity vector listed, **A**.

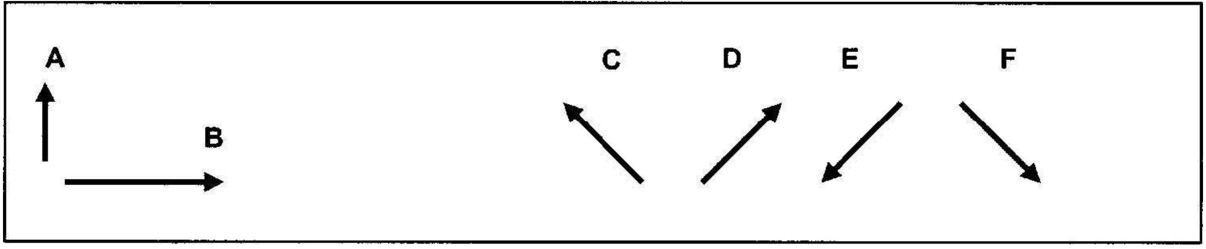


Student Response	**Pre Test**	**Post Test**
B	0%	0%
C	9%	6%
D	9%	3%
E	5%	2%
F	**66%**	**94%**
G	22%	17%
H (none)	20%	3%

Despite what this data shows, the one-on-one interviews with students found that when probed to provide an explanation or rationale for their answer, all the students interviewed who initially put G, changed their mind. Again, through the interviews, I found that the students relied heavily upon the grid and used concepts such as "slope" frequently to explain their answers.

From questions 2 and 3, it is apparent that students come in with an understanding of what magnitude and direction are.

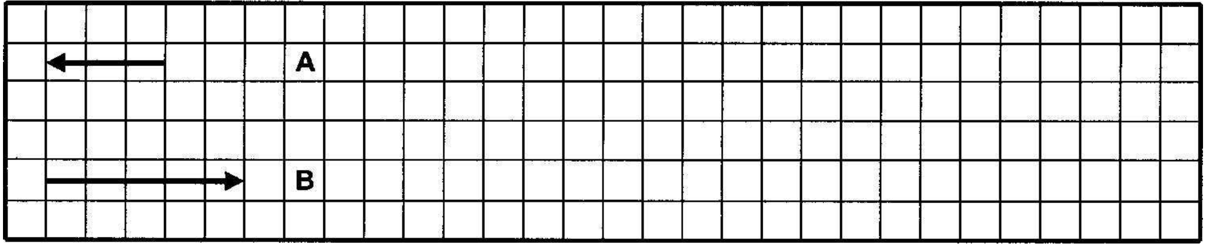
4. Below are shown two force vectors, **A** and **B**, which represent pulls on a box. If both forces act at the **same time**, which arrow (**C**, **D**, **E**, **F**) best shows the **direction** for this total force (**A** + **B**)?



Student Response	Pre Test	Post Test
C	3%	2%
D	**86%**	**86%**
E	9%	3%
F	2%	9%

There does not appear to be any effect on the performance of students on this question. Students performed as well before in the pre-test (86%) as they did in the post-test (86%). There was a slight increase in the number of students that answered “F,” possibly due to attempts to incorporate vector addition lessons that they saw in class (where you connect ends) to this one where they incorrectly connected tip to tip.

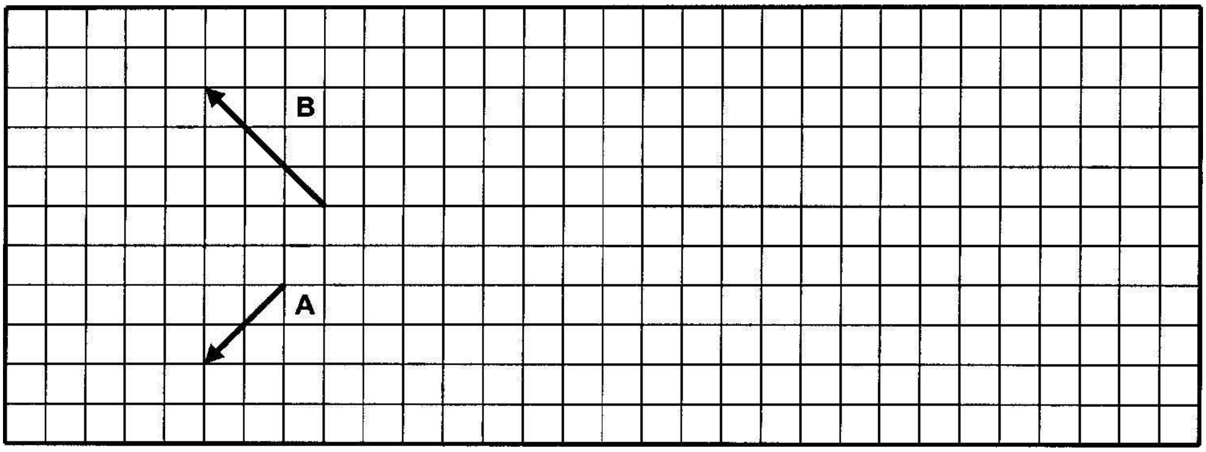
5. In the space to the right on the grid, draw vector **R** where **R** = **A** + **B**. Clearly label it as the vector **R**. Explain your work.



Student Response	Pre Test	Post Test
All Correct (Magnitude & direction)	48%	89%
Adding up just the lengths (ignore dir)	23%	5%
Arrows on both ends	15%	3%

This shows a significant improvement due to the vector unit with regards to one dimensional vector addition. The two incorrect responses reported above go hand-in-hand. Students often would place **A** and **B** end to end and redraw the vector by tracing over the combination giving a length that is the sum of the two lengths and a direction which is both of the directions. After the first day of lessons, through the student interviews, I found that the students who got this incorrect on the pre-test were able to correctly explain the correct answer. Many students resorted to a positive and negative connotation to aid in their understanding that one “subtracted” from the other.

6. In the figure below, there are two vectors **A** and **B**. Draw a vector **R** that is the sum of the two, (i.e. **R** = **A** + **B**). Clearly label the resultant vector as **R**.

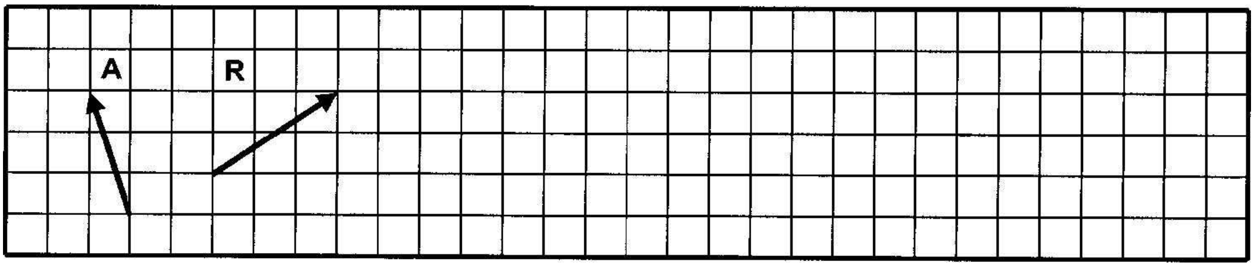


Student Response	**Pre Test**	**Post Test**
All Correct (Magnitude & direction)	**6%**	**48%**
Correct magnitude, direction is 180°	0%	12%
Subtract the magnitudes (X and Y)	23%	5%
Connect tip to tip (no rearrangement)	0%	23%

This question probed at a free vector analysis requiring the student to rearrange the vectors. While the two vectors are at a right angle to each other, they are also skewed to prevent simple component addition like in question 4. About half of the population got the previous question correct on the pre-test using intuition and subtracting lengths. This strategy was attempted here thinking that the two vectors are "opposite" of each other resulting in the subtraction of lengths in both the horizontal and vertical direction. This yielded an arrow that stretched across one square diagonally. While 23% of the freshmen did this, it is interesting to note that 31% of the students in my AP Physics class answered this way also.

While it is clear that there was a significant improvement, only 60% of the students got this question correct, or partially correct, on the post test (48% all correct, 12% correct except for the arrowhead pointing in the opposite direction). The addition of the force activity before worksheet 4 was intended to have students focus on the "effects" of the vectors and use this experience as reference. Unfortunately, this wasn't completely effective for half the population.

7. In the figure below, a vector **R** is shown that is the *net resultant* of the two other vectors **A** and **B** (i.e. **R** = **A** + **B**). Vector **A** is given. Find the vector **B** that when added to **A** produces **R**; clearly label it **B**. **DO NOT** try to combine or add **A** and **R** directly together! Briefly explain your answer.



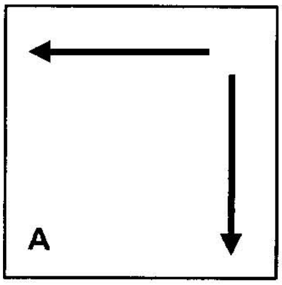
Student Response	**Pre Test**	**Post Test**
All correct (magnitude and direction)	**8%**	**60%**
Partial correct	9%	13%
Only horizontal length addition (ignore vertical)	20%	9%

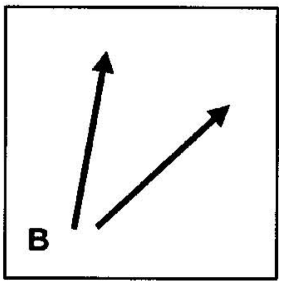
Next to question 6 and 11, this question was initially one of the harder questions for the students. This question was intended for students to deal with vector subtraction by reversing the direction of **A** and adding it to **R**. However, very few students on the post-test actually did this. Mostly from student written explanations on the test, it appears that students simply did the “fill in the missing part” exercise by drawing the beginning (**A**) and the end (**R**) and connecting the tips to complete the triangle. One concern with this method is that students may interpret the missing side as the resultant, leading to the incorrect strategy to connect tip to tip to find the resultant. In the previous question, 23% of the students did exactly this to find the resultant.

The “partial correct” indicates either the direction was flipped or not present, or the magnitude of the vector was off slightly. The most common error was ignoring the fact that **A** leans to the left and making vector **B** account for only the magnitude of **R**.

In order to effectively solve this problem, the student needed to be able to freely move either vector **A** or **R** to bring them together. In spite of the imposed grid system (that students could have used to break each down into the x and y components), most students demonstrated a free-vector manipulation through a vector addition diagram.

8. In the boxes below are two pairs of force vectors, pair A and pair B. (All of the force vectors shown have the same strength.) Consider the ***strength*** of the ***resulting force*** when each pair of force vectors acts at the same time (add up). Is the **strength** of the resulting force of pair A ***larger than***, ***smaller than***, or ***equal to*** the **strength** of the resulting force of pair B? Write an explanation justifying this conclusion.





Student Response	Pre Test	Post Test
A < B (correct response)	**42%**	**63%**
A > B	22%	22%
A = B	22%	15%
Correct identification of 3 unit vectors	9%	3%
Separation of horizontal and vertical	9%	5%

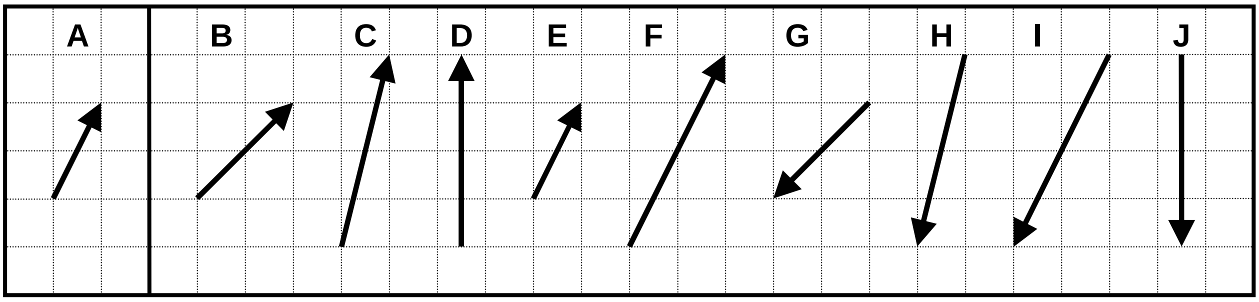
This question was a qualitative one that challenged students' simplistic way of viewing the situation of bigger (space or angle) is better. As noticed by the relatively small gain (42% to 63%), students have their own belief of how this works, with about half having a correct belief, but it may be that the intuition for this concept is deep rooted and challenging to correct. This is evident during the interview with the student from question 2, when asked about this question after we started the vector unit, she said:

> "I didn't know how to do this on the test. But we just went over this today in class. A is smaller because it takes up more space."

This was common with many of the students that initially got this question incorrect on the pre-test when asked during the interviews.

While the idea of "angle," "direction," and "closeness" may seem like a correct explanation for why A is smaller then B, I found that students used these same arguments for explaining their incorrect answers. For instance, the explanation of closeness of B's arrows or the spread of A's arrows was used for 27% of the students that got this question correct and 41% of the students who said A was bigger than B (incorrect).

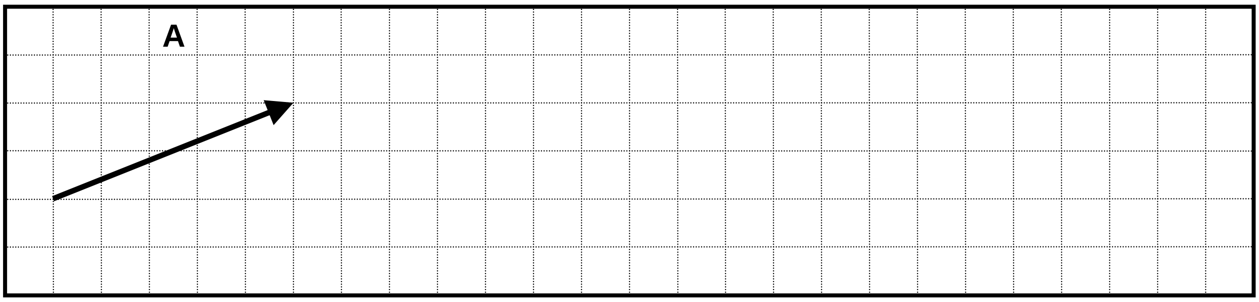
9. Vector **A** represents the acceleration of an object as a result of a constant push. If you double the strength of this push causing the acceleration to also double, which vector best represents this new acceleration?



Student Response	Pre Test	Post Test
C	14%	6%
F	**83%**	**92%**
B, D, E, G, H, I, J	<2%	<2%

One interesting note is that when I gave this to my AP Physics C classes (a calculus based class where most students have had three years of high school math and science), over 1/3 of the students chose C and less than 62% chose F.

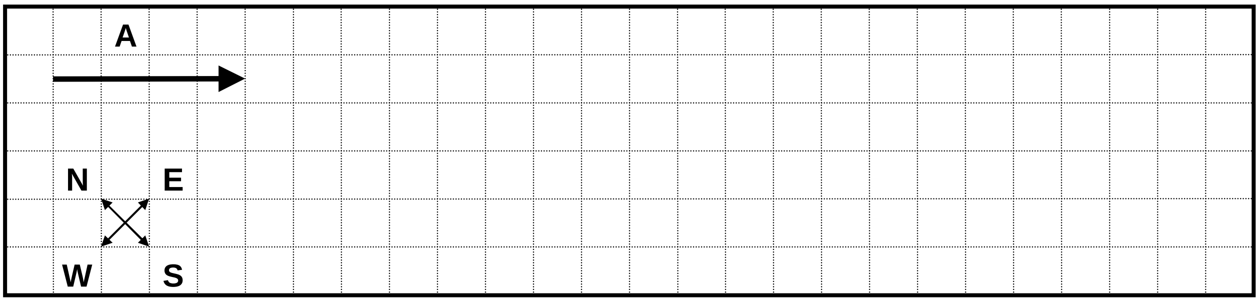
10. Vector **A** represents the displacement of a recent trip. On the right side of the grid, draw in the x and y vector components that make up this displacement vector. Clearly label each component vector as **X** and **Y** respectively. Explain your answer.



Student Response	Pre Test	Extended Post Test
All Correct (Magnitude & direction)	11%	68%
Correct X-component (mag & dir)	17%	68%
Correct Y-component (mag & dir)	15%	70%

This is one of the skills that we would hope students would be able to develop in preparation for two dimensional motion, momentum, and forces. Although this is not a "free vector" manipulation because we superimpose coordinate axes and grid on our setup, we felt that this is one skill that is critical in physics. This skill, as noted by the 11% on the pre-test, indicates that this is not something intuitive, but must be introduced and adopted. The 68% post-test average, while a significant improvement, was only 68%. One small error that came up was the flipping of the directions of the x and y components. When I talked to one of the students regarding their answer, they responded that they were trying to “create a continuous path.”

11. Vector **A** represents your velocity which seems to be horizontal. However, you mistakenly drew Vector **A** on a piece of paper that was not aligned with the normal North-South-East-West directions. On the right of the grid, draw in the East-West vector component of your velocity. Clearly label this vector as **EW**. Briefly explain your answer.



Student Response	**Pre Test**	**Post Test**
All Correct (Magnitude & direction)	2%	5%
Correct Magnitude	5%	8%
Correct Direction	20%	65%
Rotation of **A** (same magnitude as **A**)	9%	38%
Rotation of **A** (larger magnitude than **A**)	46%	40%
Direction is parallel to EW axis (either 180° to correct direction or no arrowhead)	25%	18%
Direction is perpendicular to EW axis	9%	2%
Arrowhead on both ends of vector	18%	9%

This question asked the students to realign their coordinate axes from what they would consider normal (horizontal and vertical, or aligned with the grid) to a rotated compass. However, in talking and interviewing students during and after the test, they often expressed confusion, especially over the wording, and what this question was asking for. Admittedly, this question needs to be reworded or discarded and possibly replaced with a question to get at this concept.

Almost all the students did not get this question correct. More than half of the students were able to get the correct direction, but usually by interpreting the question as completely rotating vector **A** to align with the EW axis. Most of these students interpreted the magnitude of this vector as “four squares,” not distinguishing that the length of the diagonal of the square is not the same as the length of the side. Thus, students drew in arrows that went through four complete squares diagonally which technically resulted in a magnitude increase from 4 squares to 5.7 squares.

Field Test Report – Elizabeth A. Prause

WILLIAM TENNENT HIGH SCHOOL

William Tennent High School (WTHS) is a pubic school in Bucks County, a suburb of Philadelphia, PA that serves over 2000 students. Approximately 86% of the graduating class is moving on to higher education. WTHS is a Federally-funded Title I school. During the study, WTHS was on a 4x4 block schedule. Students attended 4 classes a day, each lasting 90 minutes, for 18 weeks. Under this schedule, students needed 4 science credits to graduate. Physics is an introductory, one credit course which meets every day for 18 weeks.

The vector unit was tested during the first month of each semester. Six classes at WTHS participated in the research study: 4 Junior/Senior Academic Physics classes and 2 Junior/Senior Honors Physics classes. Students are able to self-select either honors or academic level. Class sizes ranged between 16 and 26 students.

Most of the Academic Physics students have previously taken or were concurrently taking Algebra I or a higher math class. Most of the Honors Physics students had previously taken Algebra II and many were concurrently enrolled in Calculus class. Despite the math courses taken, there appeared to be a wide range of math abilities and skills in each of the classes.

METHODS

During the study, I used the sequence outlined in the proposal, but allowed more time for my students as I felt necessary. For the honors classes, I gave them additional materials due to their higher academic level. In each case, the unit lasted about 8 or 9 days including the post-test. The underlined portions of my schedule below indicate additional material I used with my students during the Vector Unit.

My Schedule- Honors Classes (2 classes, Spring semester)

Prior to Day 1: Administer Vector Concept Quiz (VCQ) after students choose pseudonyms to use during the study.

Day 1: Do the map instruction paradigm lab and whiteboard their process and representations of displacement. Introduce the graphical representation of a vector. Begin Vector Worksheet 1 in class and finish for homework.

Day 2: Whiteboard selected homework problems. Discuss the effects of a scalar multiple on vectors. Discuss vector addition. Do colored pencil activity to introduce the commutative property of vector addition and free vector properties.

Day 3: Review Colored Pencil Activity. Trigonometry review. Begin Worksheet 2 as extension of vector addition and subtraction. Complete worksheet 2 & 3b and trig review for homework.

Day 4: Review Trig. Whiteboard homework problems. .

Day 5: Simple and Component Vector worksheets in groups and whiteboard. Vector Addition WS and text problems for homework

Day 6: Whiteboard selected homework problems. Trig Quiz. Textbook problems for homework.

Day 7: Whiteboard selected homework problems. Start worksheet 4 in class and finish for homework.

Day 8: Whiteboard selected homework problems. Do free-vector activity and whiteboard findings. Discussion of rotating axis.

Day 9: Vector Test and VCQ post-test.

After Day 9: (After completion of mechanics unit) VCQ extended post-test.

My Schedule- Academic Classes (4 classes, 3 Fall and 1 Spring semester)
Prior to Day 1: Administer Vector Concept Quiz (VCQ) after students choose pseudonyms to use during the study.

Day 1: Do the map instruction paradigm lab and whiteboard their process and representations of displacement. Introduce the graphical representation of a vector. Begin Vector Worksheet 1 in class and finish for homework.

Day 2: Whiteboard selected homework problems. Discuss the effects of a scalar multiple on vectors. Discuss vector addition. Do colored pencil activity to introduce the commutative property of vector addition and free vector properties.

Day 3: Review Colored Pencil Activity. Trigonometry review. Begin Worksheet 2 as extension of vector addition and subtraction. Complete worksheet 2 and trig review for homework.

Day 4: Review Trig. Whiteboard homework problems. Discuss vector components. Set up **worksheet 3b** and finish for homework.

Day 5: Whiteboard selected homework problems.

Day 6: Do free-vector activity and whiteboard findings. Discussion of rotating axis. Trig Quiz. Worksheet 4 for homework.

Day 7: Whiteboard selected homework problems.

Day 8: Vector Quiz and VCQ post-test.

After Day 8: (After completion of mechanics unit) VCQ extended post-test.

DATA: see Appendix E

ANALYSIS AND CRITIQUE OF FINDINGS

The vector unit was tested with 6 classes totaling 131 students. Only 100 of those students are included in the analysis. The other 31 students either asked not to be part of the study or did not participate in all or part of the study. The following table breaks down the number of students in each class that participated in the study. The letters A or H stand for Academic or Honors. The first number states which semester the class was held and the last number gives the class period.

Class	Number of Students
A11	24
A12	13
A14	13
A21	17
H22	14
H24	19
Total	100

VECTOR CONCEPT QUIZ

The Vector Concept Quiz (VCQ) (see Appendix B) was given prior to the vector unit, at the conclusion of the unit and then again 3 or 4 months after the unit. Statistically, there was a significant gain from the pre-test to the post-test. There was an average gain for the Academic classes of 28.8% and a gain of 35.1% for the Honors classes. All but two students did better on the post-test than the pre-test. This shows that the unit did have an effect on the students.

We also decided to give an extended post test to gauge the amount of retention the students had regarding vectors and vector concepts. The Academic classes had a gain from post-test to extended post-test of -2.8%, while the Honors classes showed a gain of -1.9%. Statistically these changes are small, so we can say that there was no significant change from the post-test to the extended post-test. We can therefore conclude that the students retained what they learned during the unit.

The following data table shows the average percentage scores for each class on each of the three tests. As above, the letters A or H stand for Academic or Honors. The first number states which semester the class was held and the last number gives the class period. The average is not a simple average of each class, but the average of all the students since not all classes have the same number of students.

Class	% Pre	% Post	% Ext
A11	28.2	54.7	57.3
A12	40.7	67.0	59.7
A14	32.0	60.4	53.4
A21	36.6	69.5	67.9
Average	33.3	62.1	59.3

Class	% Pre	% Post	% Ext
H22	46.8	80.2	80.5
H24	48.3	83.2	80.8
Average	47.7	82.8	80.9

While analyzing the VCQ, I found that both the Honors and Academic classes' worst average score was on question #7 specifically on the post-test. This was not surprising as it dealt with subtraction of vectors which I did cover, but only very briefly. The Honors class scored an average of 61.3% on this question, whereas the Academic class scored only a 32.3%. In the future, I need to decide if subtraction of vectors is required as part of my vector unit or not. If it is, then I need to teach it more thoroughly. If I don't feel it is necessary, then question #7 should be removed from my VCQ.

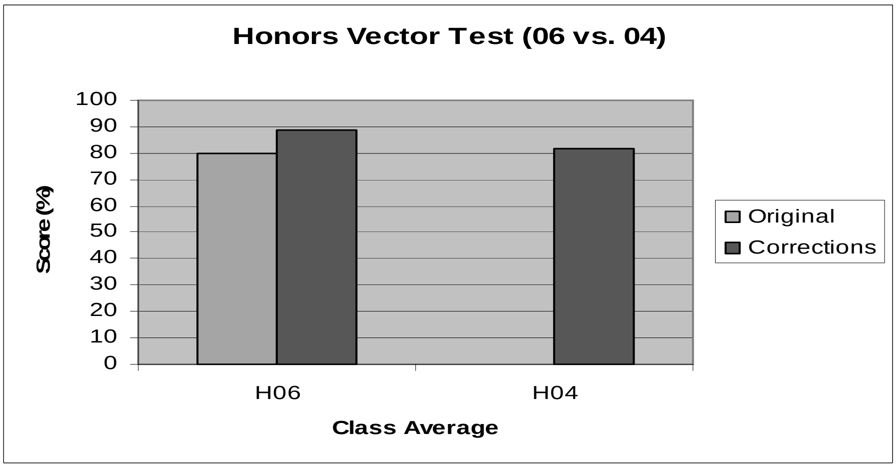
Another interesting piece of data that I found was that my Academic class' average score for question #4 was worse on the post-test than it was for either the pre- or extended post-test. It appears that 13 students had #4 incorrect on the post-test, while only 6 of those 13 had it wrong on the pre-test, and only 4 of those 13 had it wrong on the extended post-test. Of the wrong answers, letter **F** was chosen most often. For answer **F**, the students appeared to be drawing their resultant from the tip of **A** to the tip of **B** and for **C** from the tip of **B** to the tip of **A**. Choice **E** would have been the equilibrant of **A** and **B**.

Name	**Pre**	**Post**	**Ext**	**Class**
Otto Fueller	E	E	E	A11
Alexis	D	C,D	D	A11
Jaques	D	F	D	A11
David Ramanthorn	D	C	D	A11
Shadow	?	C	D	A11
Emeka Okafor	D	F	F	A12
Barry Sosa	C	C	C	A12
Bnana Maryland	D	F	F	A14
Baxter	D	F	D	A14
Shibleth	E	E	D	A14
Claire Voyant	NONE	F	D	A21
Dinah Mite	D	F	D	A21
Justin Credible	E	F	D	A21
# of Cs	1	4	1	
# of Ds	7	1	9	
# of Es	3	2	1	
# of Fs	0	7	2	

I also noticed that both the Honors and Academic classes had a decrease from the post-test to the extended post-test for questions #9 and 11. Both groups had a change of -16.7% for question #9. Question # 9 refers to graphically multiplying a vector by a scalar. This was a concept covered during the vector unit, but it is not encountered in the rest of the modeling materials. For question #11, the Honors had a change of -6.5%, while the Academic classes had a change of -13.7%. This question refers to free vectors and how to adapt to a rotated axis which we use during Newton's Laws. Despite the fact that students have used this concept, they seemed to find the question confusing. Many students told me during the each of the tests, they did not understand what the question was asking them to do. I can only conclude that many of them guessed when they answered this question.

Vector Test

This year, I gave a test to my Honors class that I also gave to my Honors class in the Spring of 2004. I decided that I wanted to see how their grades compared between the two years. In 2004, I allowed students to earn points back on their tests by retaking the test. In 2006, I allowed students to earn points back on their tests by turning in corrections. In both cases, it was an optional assignment. The class average for my 2004 class after the corrections was 81.8%. I do not have their scores before corrections as I put their new grade in overtop of their original one. The class average for 2006 before corrections was 78.4%, which is close to the 2004 class average. However, if we look at the class average after their corrections, the class average jumps to an 87.9% which is higher than the 2004 class. From this, I feel that the vector unit I did with my Honors class this year did help them learn vectors.



ANNECDOTAL EVIDENCE

In the past, when I was trying to teach vectors to my Academic class, I felt very unsuccessful. In the Fall of 2004, one student would actually get so frustrated that whenever I started discussing vectors, he would try to just put his head down. The class complained when I tried to teach them vectors and again when we saw vectors in Newton's Laws. They struggled so much with the vectors in Newton's Laws that I eventually gave up trying to do some of the inclined plane problems and any other problems that required advanced used of vectors. When it came time to projectile motion, we only dealt with horizontal projectiles because they could not handle using vectors.

In implementing this unit this 2005- 2006 school year, the students did struggle with learning vectors, but they did not seem as frustrated. They really liked the map activity and could easily see how we could use vectors to represent displacements. The worksheets challenged them to think and figure out how vectors really worked. When we reached the force unit and we were drawing force diagrams they understood that the vectors had to represent the forces. The attitude of most of the students was, "Hey, we've seen this before. We already know how to do this." Therefore, they understood that the length of the vectors on their diagrams did matter. They were also able to add up the vectors to see the direction and magnitude of the net force. They also appeared to be more comfortable with the problems. With my classes this year, I was able to teach inclined planes and level plane projectile motion (up and down) rather than just horizontal projectiles.

One of the goals of the unit was to get the students to a free vector model where the vector did not necessarily have to be attached to a grid coordinate system. I had a large three dimensional vector made out of K'NEX in my classroom. When we were discussing vectors, I would often take it off the wall and use it to ask my students questions. I would then ask them to tell me what they wanted the vector to be. They gave different answers, such as 6 ft "that way", 3 meters E or 2 Newtons S. I would move the vector around the room keeping it the same size and direction and ask my students, if it was the same vector, even though I wasn't in the same place. They understood that I didn't change anything but the vector's location. If I pressed them, they usually mentioned that we added vectors by moving them around to line them up tip to tail. The vector did not necessarily have to stay in the same spot to be considered the same vector. I would then change the direction of the vector and ask them the same question. The students usually all told me that it was a different vector because it was pointing in a different direction. I would then take part of the vector apart and ask if it was the same vector. They all knew that by changing the magnitude, you changed the vector. From these discussions, I felt that they understood the basic definition of a vector and that the vector was not tied down to any specific location, that it was free. It still represented the same quantity regardless of its location.

Field Test Report- Kathryn Morgan

Background

This is a private school with a student population from around the globe. The classes that took place include a Higher Level IB Physics class (IB), ninth grade IGCSE Physics class and two grade eight General Science classes. The IB consisted of six students. Of the six students only 1 was a native English speaker and all six had a minimum of two years of physics and were at a very high math level. The IGCSE class consisted of 16 students of varied math level and only 4 are native English speakers. The two grade eight classes were made up of 22 students each and consisted of varied levels of math and science.

Methodology

The study was preformed based on the previously outlined time line. The IB class also studied using trigonometric laws (Sine and Cosine Laws). During our unit on Orbital Motion, this class also studied the use of 3D vectors.

A second IGCSE Physics class was taught by my colleague who also teaches a vector unit. This class was given a post test to compare results. The two IGCSE Physics classes took the same Exam and had similar averages.

Analysis

In analyzing the student scores on the Vector Concept Quiz (VCQ), I broke the scores down into raw scores; percentages for each question, student, and class (see Appendix F). When looking at the gain in percentages, I noticed a significant gain noted in the table on the following page.

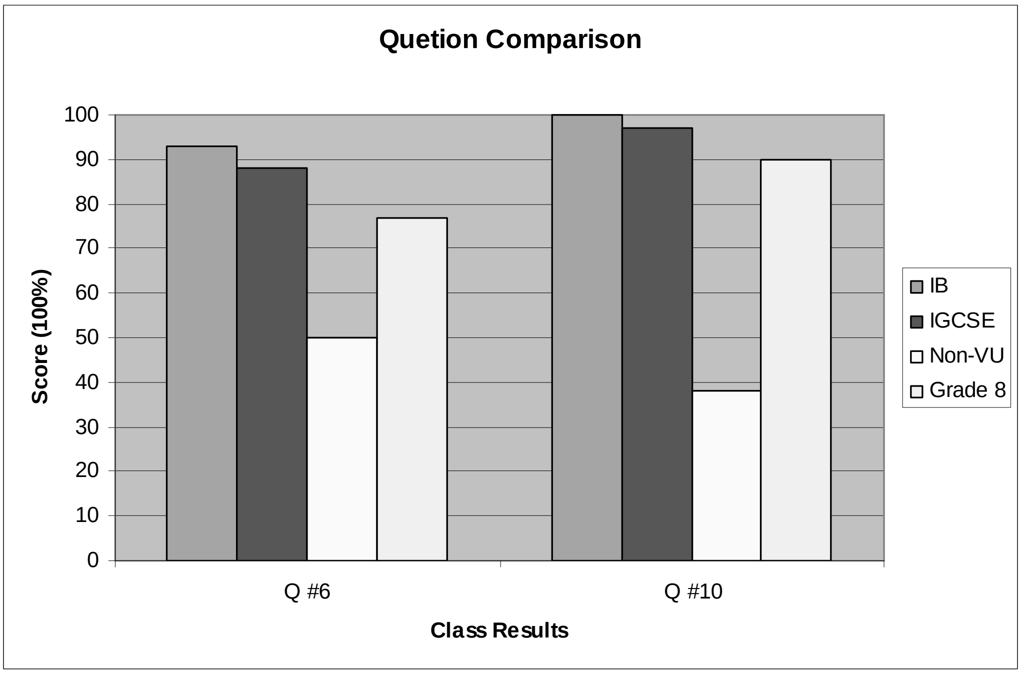
Class	Pre-Test (25)	Post-Test (25)	Percent Gain
Y12 IB	15.6	21.2	59.7%
Y10 IGCSE	7.88	17.84	58.2%
Y9	9.5	18.7	59.4%

The student gains show the vector unit we developed increased student comprehension and manipulation of vectors according to the VCQ.

There are two aspects about the pre-test scores amongst my IB students. First off the average pre-test score is 62.4% yet every student in the class had a minimum of two years experience in high school physics where they had done some work on vectors. Secondly, only two out of five students managed to receive partial credit when asked to describe what a vector is. Most of them claimed it was something they hadn't thought of before. When studying in acceleration one student even claimed that acceleration was not a vector because he had never had to include direction before. He believed objects could either speed up or slow down. When referencing our two dimensional worksheet and applying concepts learnt in the vector unit, he was able think through his justifications and modify his personal model of acceleration. He has taken three years of physics before this class, and had never understood the concept of acceleration as a vector until this year.

I administered the extended post test to the IB and IGCSE classes at the very end of the school year. After looking at the extended post test scores, the students had an average gain of an additional 10%, or 2.5 points on our 25 point quiz. These numbers show the students were able to improve their understanding over the course of mechanics and retain the knowledge through out the other units we covered.

When comparing my IGCSE Physics class to class who completed a different vector unit, I found no significant difference. There was a wider variety of levels amongst the second class which led to a much lower mean (66%) and a larger standard deviation. When I broke down the individual scores on each question, I found something interesting. There were two questions on the test that my students significantly outscored the second class. My grade eight General Science students were even significantly higher on these two questions than the second physics class.



Question #6 involved adding two ‘free’ vectors and reporting the resulting vectors. This is considered the highest level of vector comprehension according to Poynter (Poynter & Tall, 2005a). My classes were able to produce the correct answer to an average of 87% accuracy level.
The second question that I noticed a significant difference is question #10. This question involves taking a vector and breaking it down into x and y components. My students outscored the other physics class by 60%. I also had some students in my grade 9 and IB classes who drew the x axis along the given vector and the y-axis perpendicular to the x-axis instead of the conventional up and down relative to the page.

The questions the students had the most difficulty with in the post test and extended post test was questions number 3 for my IGCSE students and question 11 for all classes.

When reviewing results for question 3, I noticed that all but two students in the class at least received partial credit. The question asks to identify all vectors that have the same direction as the one given. They were able to identify the correct vector; however most of them also included vectors that were in similar direction, but an incorrect answer. I believe their reasoning is because the most common wrong answer is the same length as the example given and so they thought it would be in the same direction. To improve this, I will have to emphasize exact directions instead of general directions.

After reviewing question number 11, which asks the student to identify North and South components on a grid where the directions are given on an angle shifted 45degrees from the conventional up and down. Common answers were pointing the arrow West instead of East or answers that were completely wrong and statements of ‘I don’t know’. I believe part of the problem with this question is in the wording. It is not clear what the students are being asked to do and with the grid in the background, it almost sets them up to fail on this question. I think this question should be modified or removed.

Comparing these four classes to my previous years in teaching physics is difficult because I did not have students at similar levels than in the past. I did find my grade nine students had fewer problems in drawing force and momentum diagrams compared to previous classes. They were also able to solve two dimensional problems using scale drawings.

If I were to continue this study, I would like to include more instruction on three dimensional vectors with my advanced students. Despite their full confidence in working with vectors in two dimensions, my IB students had a lot of difficulty applying the same vector properties to three dimensional situations. In discussing their issues with manipulating 3D vectors they explained that they are 'just not the same'. They also mentioned that it was difficult to picture things in 3D. I then reminded them we live in a three dimensional world and they seemed stumped. We struggle through some problems and representations and I feel the most effective was looking at the vectors on a three dimensional visualizing program, VPython. I was very interested in why it was so difficult for them, and how we could adapt our unit to help overcome these barriers.

Conclusion

At the end of this study, we hoped that we will have a coherent and contextual introductory unit on vectors that will help students establish an experiential basis for their comprehension and manipulation of free vectors. We also hoped that by focusing on the "effects" of vectors (Poynter & Tall, 2005a) that students will have some conceptual resource to draw upon when presented with vectors quantities and manipulations in future units such as velocity and acceleration (in motion maps) and forces (with force diagrams).

We have concluded from our analysis of the VCQ that our vector unit is effective in increasing student understanding and manipulation of vectors. With a sample size of 201 students we preformed a Directional Paired Sample t-test and we report a significant increase between the pre-test and post-test scores with 95% confidence. When a Directional Paired Sample t-test was done between the post-test and extended post-test scores, we report with 95% confidence there is no significant difference. What these statistics tell us is that the students gained understanding of vectors and were able to retain their understanding over the course of the year.

Implications for Further Study

After our vector unit, we decided that some improvements and changes should be made. Our VCQ seems to be too dependent on graphical representations of vectors. Our assessment tool should be modified to include graphical and mathematical representations to test student understanding. The VCQ does not look at the effect of vectors, but only at the resultants. We would like to see the students have a better understanding of what the resultant represents, i.e. the effect of the original vectors.

Due to uncontrollable circumstances, we were unable to obtain a control group. In future studies, a control group would help determine the effectiveness of our unit compared to others.

This unit could also be enhanced for upper level students by including vectors in three dimensions. The assessment would have to be modified to test this. The vector unit could also be more integrated into the curriculum rather than as a separate unit, based upon teacher preference. For instance, introduction of one dimensional vector analysis could be introduced in the constant velocity and acceleration units as students need it. Similarly, the two dimensional aspects could be introduced during forces in the study of force diagrams, inclined plane problems, and sum of forces equations.

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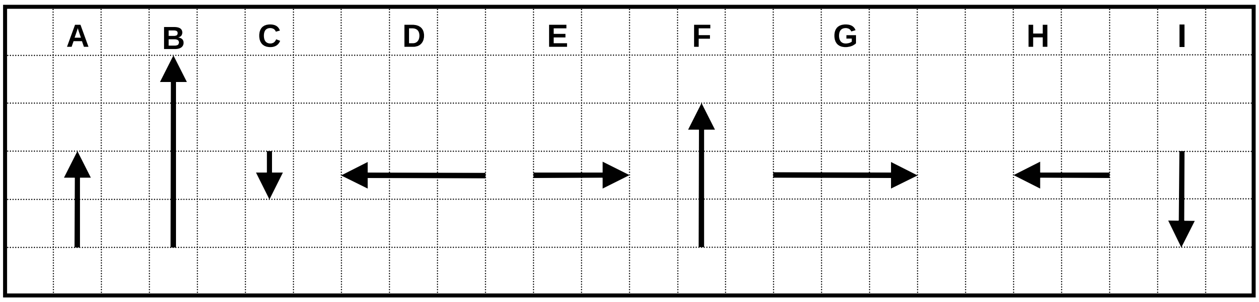
Appendix B

Vector Concept Quiz

(Modified and used with permission from Nguyen & Meltzer's Vector Concept Quiz)

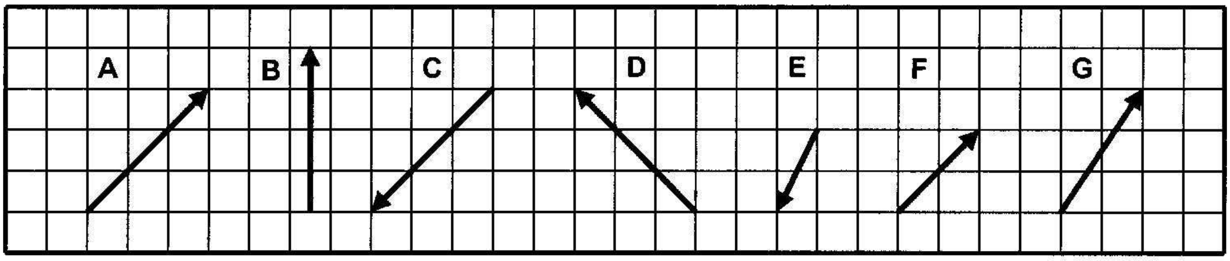
1. Write a brief description of what a vector is:

12. Consider the list below of pushes that you exert on a box and write down **<u>all</u>** pushes that have the same **<u>strength</u>** as each other. For instance, if push **W** and **X** had the same strength, and the pushes **Y**, **Z**, and **A** had the same strength as each other (but different from **W** and **X**) then you should write the following *W* = *X*, *Y* = *Z* = *A*.



Answer:______________________________

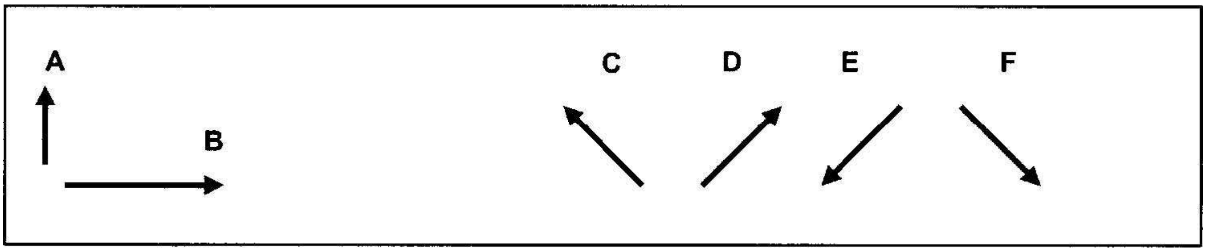
13. List all the displacement vectors that have the same **<u>direction</u>** as the first displacement vector listed, **A**. Please explain your answer.



Answer:______________________________

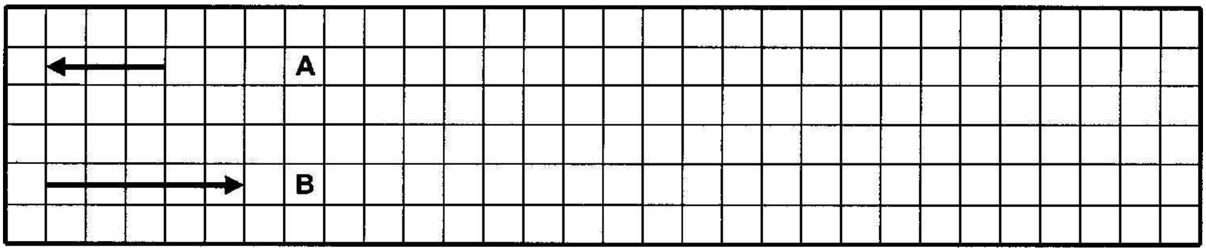
Explanation:

14. Below are shown two force vectors, **A** and **B**, which represent pulls on a box. If both forces act at the **same time**, which arrow (**C**, **D**, **E**, **F**) best shows the **direction** for this total force (**A** + **B**)?



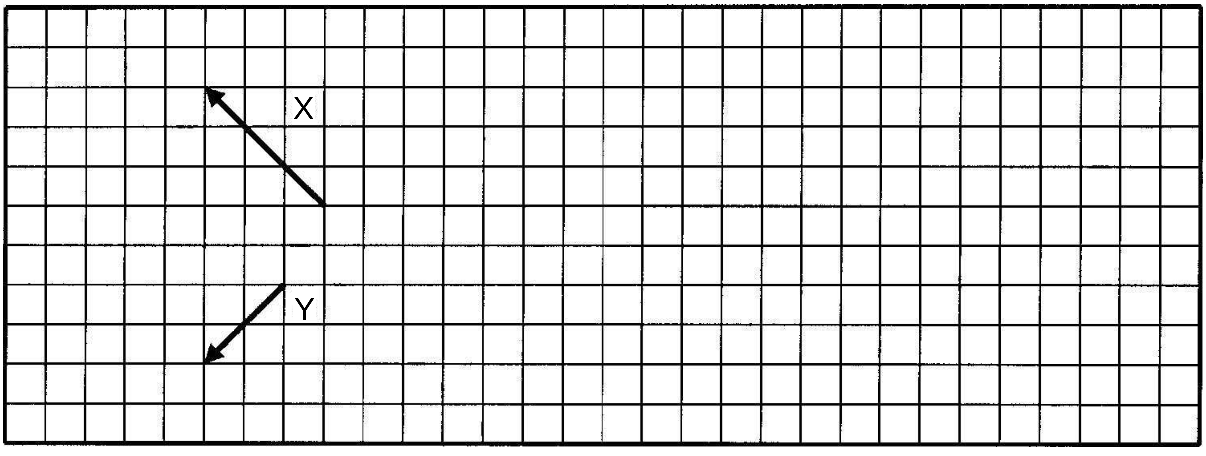
Answer:______________________________

15. In the space to the right on the grid, draw vector **R** where **R** = **A** + **B**. Clearly label it as the vector **R**. Explain your work.

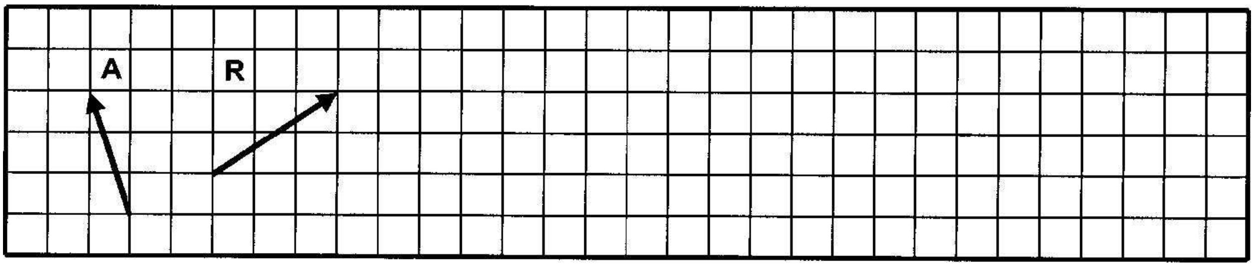


Explain:

16. In the figure below, there are two vectors **X** and **Y**. Draw a vector **Z** that is the sum of the two, (i.e. **Z** = **X** + **Y**). Clearly label the resultant vector as **Z**.

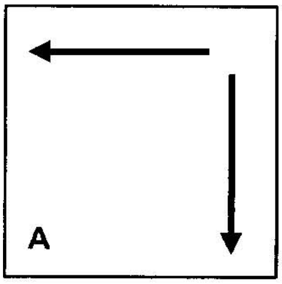


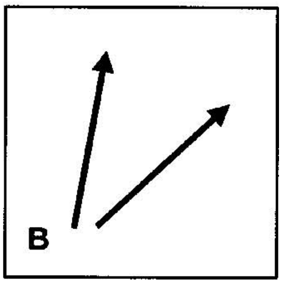
17. In the figure below, a vector **R** is shown that is the *net resultant* of the two other vectors **A** and **B** (i.e. **R** = **A** + **B**). Vector **A** is given. Find the vector **B** that when added to **A** produces **R**; clearly label it **B**. **DO NOT** try to combine or add **A** and **R** directly together! Briefly explain your answer.



Explain:

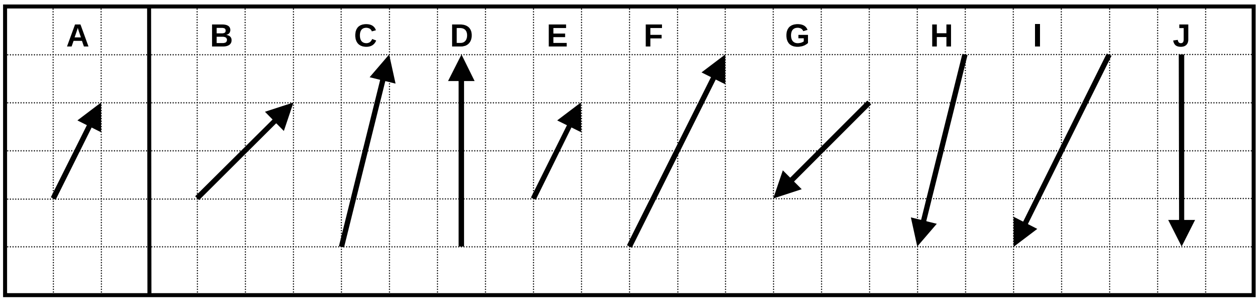
18. In the boxes below are two pairs of force vectors, pair A and pair B. (All of the force vectors shown have the same strength.) Consider the ***strength*** of the ***resulting force*** when each pair of force vectors acts at the same time (add up). Is the **strength** of the resulting force of pair A ***larger than***, ***smaller than***, or ***equal to*** the **strength** of the resulting force of pair B? Write an explanation justifying this conclusion.





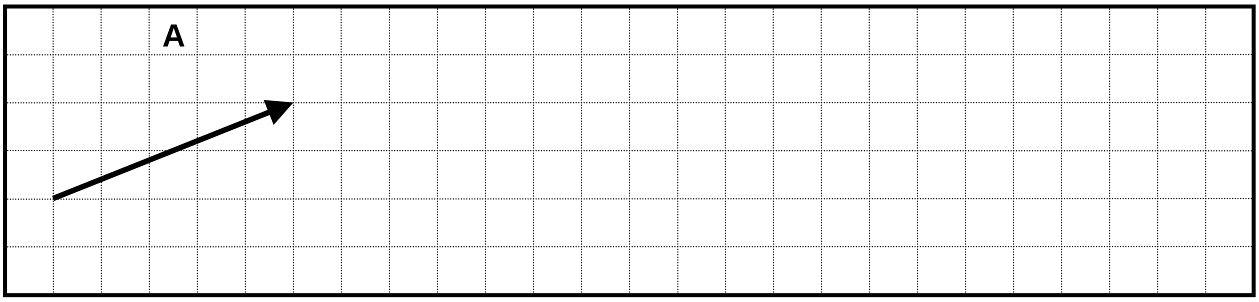
Explain:

19. Vector **A** represents the acceleration of an object as a result of a constant push. If you double the strength of this push causing the acceleration to also double, which vector best represents this new acceleration?



Answer:______________________________

20. Vector **A** represents the displacement of a recent trip. On the right side of the grid, draw in the x and y vector components that make up this displacement vector. Clearly label each component vector as **X** and **Y** respectively. Explain your answer.



Explain:

21. Vector **A** represents your velocity which seems to be horizontal. However, you mistakenly drew Vector **A** on a piece of paper that was not aligned with the normal North-South-East-West directions. On the right of the box, draw in the East-West vector component of your velocity. Clearly label this vector as **EW**. Briefly explain your answer.

Explain: