COMPILATION: **Cognitive Instruction in Math Modeling (CIMM), used in chemistry**

A chemistry modeler posted: "I had more trouble this past year with students not really understanding complex units. When we used the gas law constant 0.0821 L-atm/mol-K too many treated the units as symbols, inserted numbers and included them in their calculations. Given that I show them how to rearrange PV=nRT into R=PV/nT and then solve for R in various units...I'd think that understanding would happen.

Then again, I find it odd that they can answer conceptual questions about the meaning of driving at 60 miles per hour but can't use that same logic when discussing density or molarity or any other simple 'per' quantity."

Joe Morin, a physics modeler, added: "Even some of my physics students, who already have had chemistry, mix up the variables and the units. They might be thrown off by the word "per" in miles per hour. Teachers need to be very explicit in explaining the variable, the number, and the units associated with the number. I think it helps to explicitly (and very frequently) break down a numerical answer by dividing by one. For example 80 mph is 80 miles per hour, which is a distance of 80 miles divided by 1 hour, which means the object would move a distance of 80 miles during a time of 1 hour (if it continued moving at that speed for the entire hour) ... or in 1 hour the object would move 80 miles. Then ask questions: how far would the object travel in 2 hours, a half hour, etc.; how long would it take to go 40 miles, 160 miles, etc.

I really think teachers need to do this whenever a new set of units is introduced. When I first started teaching chemistry and physics, I assumed that students actually learned some math in their algebra courses; but really all they have learned is to do meaningless repetitive algorithms one after another until they pass all the tests. In science, we need to start from scratch and teach the math so that it has meaning and develop the understanding that is missing in our students' minds.

Date: Fri, 11 Jul 2014
From: Rob MacDuff

Joe Morin wrote the following:
> When I first started teaching chemistry and physics, I assumed
> that students actually learned some math in their algebra courses; but
> really all they have learned is to do meaningless repetitive algorithms one
> after another until they pass all the tests. In science, we need to start
> from scratch and teach the math so that it has meaning and develop the
> understanding that is missing in our students' minds.

I could not agree more with Joe on this. Starting out the year with a unit on math seems necessary. *Numbers in math classes are taught as nouns, however, in science they are all adjectives.*

I have a blog, not totally finished, where these issues are discussed in detail. We are working on such a math unit that would sit in front of a physics or chemistry course. The website is TRU-ED.org. Any feedback would be welcomed.

Date: Mon, 14 Jul 2014
From: Brenda Royce
Subject: Re: algebraic or analogical reasoning/dual meaning

Rob MacDuff wrote:
Students come into your class with underdeveloped metaphorical, analogical and transitive reasoning skills-- skills with which they are born, but not taught to utilize in schools. They also come with varying degrees of algebraic reasoning skills which depend heavily on memorization, which are taught in schools. The visualization case requires students to reason analogically; they will not recognize it as such nor be able to articulate it, but yet they do it. Thus the reason the error disappears is that you have switched their reasoning from algebraic to analogical. Coordination enhances both.

I attempted to respond to this thread Saturday and exceeded the line limit, so it didn't post. I'll be more concise this time - especially since Rob has made most of my points already. I was struggling with the "kids don't get the math" issues when I began encountering Rob's ideas about math. I finally was able to take his CIMM class about 3 years ago and began attempting to apply the tools to my chemistry class. I also read a book he recommended (Brian Butterworth's "What Counts..." - got in Amazon for less than $5).

What Rob and Butterworth described began giving meaning and language to the problems I saw - that students weren't actually reasoning with their math because they had a disconnect between the reasoning they did natively and the algebraic symbols and "procedures" they used. They weren't even using the same parts of their brain when "doing math" as they did when they solved accessible quantitative problems. They tended to see all "numbers" in math as being essentially equal in meaning and function. We science folks get that there are differences between 2, 2 g and 2 g/mL because we know the context and meaning of them in a practical way - but a number of our students don't make the connection - it's all "2" with other annoying stuff attached that makes it harder to "do" their math procedures. In chemistry, I can get nearly every student to answer simple stoichiometry questions they can reason in their heads with confidence, but as soon as they must resort to calculations they cannot "do" in their heads, many cannot model their reasoning on paper - the algebraic symbolism has become disconnected from the analogical reasoning they did fluidly.

As I learned from CIMM ways to give them tools to understand what the calculations they are doing *mean* (for chem it's nearly all simple proportional reasoning), many of the problems these posts have described have begun to disappear. Unit labels do not get confused with variables nearly as often. The role of unit conversion factors and relationships between quantities in a system (like density, heat of fusion, molar mass, mole ratios in a reaction...) can be distinguished from simple quantities (mass, counts, energy...). Students can stop "cancelling units" and discuss contextually the reasoning they are doing with how they set up the calculations. Yes, we have an alphabet soup of letters that have multiple meanings (m for mass or meter - one a variable, the other a unit), but I don't think that's the real problem. Our students generally navigate the same issues of ambiguity in reading - all those letters! but they learn to understand them in context of what they are combined with because they understand what they *refer to* when they see them in sentences.

Our struggling math students did not learn to do that with algebraic symbols. *Rob's intermediate language of dots activates their native analogical reasoning.* It gives them a way to model their thinking so they can reflect on it and allowed me to make the distinction between number, quantity, and ratio relationships - and their roles in understanding a physical system - in a meaningful way. I have never been very successful at doing that in the context of normal algebraic symbols because, as I realized from Rob, the algebraic symbols are often not activating the right parts of the brain. We have to actively help them build bridges between all the parts of the brain needed to do the reasoning in symbolic language.

If you get a chance to explore CIMM (and read Butterworth), I would recommend it.

Date: Thu, 17 Jul 2014
From: Brenda Royce

Two people asked me to follow up my post.

I would not presume that I have delved the depths of the problem - only that I have begun to realize more facets of the problem than I had before. The things that have helped me and my students are a more refined understanding of how students perceive numerical information ("numbers"), as well as how they perceive what they "do with numbers" ("math") and what is/is not happening in their minds with they "do" math.

So, my first insight: *We reason natively in quantities, not numbers.* The fact that students resist putting units on various quantities is likely because they see "numbers" as objects (nouns) that can be manipulated by math rules to give you answers. Units get in the way. Yet our minds reason in quantities - where numbers are adjectives describing an object (3 apples).

Second insight: When "doing math" students do not keep the meaning of the quantities in play. The numbers stop being connected to the thing they are describing and become the object they are thinking about. But there are meaningful differences between the ideas of number, quantity, unit, and relationships (for chem there are a number of ratio relationships we use; there are other kinds of relationships, too) - and we might add "variable". Students often have poor differentiation of these ideas, and so don't get why we make such a big deal about units. Insight 1.5 - MY explicit understanding of these had room for more clarity, especially regarding what a number is. *Rob's CIMM representations of quantities (dots) and the quantitative structure within a system (or "whole", "totality") gave me the tools to be more explicit, and to more readily differentiate these ideas for my students.* Here's a very brief explanation:

Unit: (the size of 1) represented in its simplest way by a "dot" - O
Quantity: represented by a group of dots - OOOOO

These two are related to one another as Quantity = number * unit.

So how would we represent a number? How do we know the quantity 5 dots requires a 5? Because the relationship between the quantity and its unit is OOOOO/O, which we call 5. Five is also represented by (OO)(OO)(OO)(OO)(OO)/(OO) where (OO) is the unit (NOT conceptually trivial). Numbers are relationships (Q/u), not things. They give us the "scale" of the unit (think unit vector vs vector). Many rich implications here I do not have space to go into.

Relationships exist between parts of a system. For chem they are typically a ratio of two quantities both describing the system. Ex: 5g/mL, or OOOOO/@. They are a property (permanent or temporary) of the system. Relationships function by logically transforming one quantity into the related quantity.
Q1 * R = Q2

Third insight: Many students can do math without activating the part of their brain that does quantitative reasoning. It is possible to accurately carry out math manipulations only using the parts of our brain that
1) recognizes symbols (give them names, not meaning - that's another part of the brain - like me "reading" Spanish aloud without real understanding), and
2) recalls procedures or math facts. Since the students have often disconnected the math from anything 'real', having a quantity-based visual "language" (dots and groupings) for expressing quantitative ideas and reasoning lets us put the reasoning back into the algebraic expressions. By modeling the thinking we are needing to do in a non-algebraic way first, and discussing the reasoning it expresses, I can help students represent the reasoning they understand but can't do in their heads. For example:

For 2A + B --> 3C; If you have 4 moles A, how much B can react?
Let O = 1molA and & = 1 molB.
The system is known to contain [OOOO] and an unknown amount of (&). Our reaction tells us that each (OO) requires (&), or (&) for every (OO). It is not hard to see we need to find how many (OO) there are in [OOOO], and react [&] with each (OO) we find.

So we could represent that reasoning as: [OOOO] --> [(OO)(OO)] --> [&&].

More formally it can be written [OOOO] * &/OO = [(OO)(OO)] * &/(OO) = [&&].

This could also be written as [(OO)(OO)]/(OO) * &. Do you see the number 2? This is telling us how to rescale (&) to be able to react in a system containing (OOOO).

The amount of A was regrouped ("divided") into two groups of (OO), and then each (OO) was associated with (&). When I'm working with students who are struggling, I have them draw a dot diagram of the system and do the regrouping. When I ask them to explain what they did, they will often say they "divided" it into groups, indicating they understand division but don't think about it when doing math. Now I can point out that the bottom of the ratio relationship (&/OO) tells us how to regroup (divide) the initial quantity. That's why it is under the division symbol. And this is why the bottom of the relationship MUST match the quantity type and unit (O) of the initial quantity (OOOO). How can you regroup by a different kind of quantity? (How many groups of 2 dogs can you make of 10 cats?)

So, this is the reasoning represented by 4 mol A * 1 mol B/2 mol A = 2 mol B. Extending it to less mind-friendly numbers is easier when the reasoning is clear.

This alone has had HUGE pay off when my kids come to recognize its meaning. It requires that they can tell the difference between a quantity and a relationship when they encounter them (which I also work on). *When their thinking is anchored in real quantities with understandable reasoning processes, units are simply part of the information, and variables and units don't typically get confused because they both have separate, understood meaning* - same reason we don't confuse them.

Date: Fri, 18 Jul 2014
From: Jane Jackson <jane.jackson@ASU.EDU>

I love Brenda Royce's posts on CIMM yesterday. So clear! and compelling for student learning!

If you would like to learn more, please visit the CIMM webpage that I developed on the ASU modeling website. Visit
http://modeling.asu.edu
and click on "Cognitive Instruction in Mathematical Modeling" (CIMM).

Read these short documents on what CIMM is.
* The Math Problem, by Rob MacDuff, researcher/developer of CIMM (2008)
* Enter the Dots, by Richard Hewko, co-developer of CIMM
* CIMM: An Approach to Mathematics Education, by Rob MacDuff (2009)
* A sample lesson: teacher notes and worksheets on dots

I visited a 4th grade class where CIMM was being used. It was astounding, the deep level of comprehension of the students.

Note added in Feb. 2015: visit Rob MacDuff's blog (new in 2014): Cognitive Instruction in Modeling Math and Physics:
https://trueddotorg.wordpress.com/