COMPILATION: modeling without trigonometry

a listserv discussion in Jan. 1998. Updated and posted in Jan. 1999, with new comments.

In January 1999 Larry Dukerich posted to the modeling listserv:

"Jane Jackson has gone through the posts made to the listserv in 1998 and has assembled this "thread" regarding the question of how much trigonometry (if any) is required in order to use Modeling to study the force concept. As you will note, there is quite a variety of viewpoints, which should reinforce the notion that Modeling is not so much a curriculum one must follow lock-step than a curriculum DESIGN."

Date: Mon, 19 Jan 1998
From: G (a new modeler & a young teacher)

A while ago on this list I commented to a Modeler regarding how to handle physics for students who cannot do trig. I told him to teach it to them -- in essence to "raise the bar." I was wrong. That was an easy solution in September with my 2 years of physics teaching experience. Now it is January and I am 25% more experienced.

We are just finishing the semester and I have been under a deluge of students dropping like flies at an Orkin convention. They tell me *they LIKE the course, but it is too hard -- they cannot do the math and thus "turn off" most of the time.* I could point the finger at the math department and science department (both are acknowledged as the weakest in the school), but that does neither me nor the students any good.

Question - *how does one teach Unit 4 without trig? How does one teach Unit 5?*
IS THIS BEING ADDRESSED BY MODELING CENTRAL? WHAT DO HESTENES, BRAUNSCHWEIG, RICE, SWACKHAMER, DUKERICH, AND THE OTHER GURUS THINK ABOUT THIS?

I would like to hear from all of you out there, but I REALLY want to hear from the big wigs. Do any of you have any suggestions as to *how low math (low level/non-advanced level of cognition) students can learn physics via modeling?*

Date: Mon, 19 Jan 1998
From: Richard McNamara

I have taught a course that was targeted for students who had failed Algebra I or had done poorly in the math courses leading up to Alg. I. I was teamed with a math teacher and we had the kids for two periods a day for a full year. Those who passed the course received one credit in Algebra and one for conceptual physics.

We used a modeling approach and had them develop the algebraic tools the same way we developed the physical relationships. One of the labs we used a force lab based on trig and right triangles. We would hang a known mass from two strings attached to spring scales suspended from a horizontal rod. We changed the angle between the strings by increasing the distance between the points where the spring scales were connected to the rod.

In the pre-lab, we elicited the idea that each string pulled a little to the side and some up and down. We initially used graphical addition of vectors to show that the forces always added up to zero. Even when the force of each string exceeded the force of the earth on the hanging mass. We eventually introduced the idea that there may be a way to do these problems analytically instead of graphically. They began to buy the idea that math could make something easier.

The idea of 'raising the bar' can work if you're willing to go all the way. You may have to sacrifice some content, but it can be done. If you don't want to bite off this sizable chunk of material, modeling can be done if you *limit scenarios to those involving forces at right angles.* There are plenty of good solid problems that can be done with this limitation.

If they think the math is too hard, *give them problems in smaller chunks.* If it's a solution that requires multiple substitutions, *ask them for intermediate steps in the solutions process.* Ask them for alternative methods of solutions. *Ask for graphical solutions or diagrammatic solutions.* That was one of the things that impressed me the most when I was working with the students who had difficulty with math. They could use graphs and diagrams to solve problems that would take a page or two of algebra in one or two steps with a graph or diagram.

Date: Mon, 19 Jan 1998
From: John Ball
G wrote:
> A while ago on this list I commented to a Modeler regarding how to
> handle physics for students who cannot do trig. I told him to teach it
> to them -- in essence to "raise the bar." I was wrong.

No, you were right. The answer to every tough problem cannot be "lower the bar." We have done that in American education for thirty years now and have little good to show for it. The problem is confusing the ends with the means. You want students to be able to do trig based physics -- otherwise, you essentially ignore components and that limits your physics too much. How can you best introduce the trig?

The answer depends on your school. It may mean *working with your math department to ensure that trig skills are introduced early enough* to help you out. It may mean *slowly teaching the skills yourself* -- the skills they need are fairly trivial, and the students can gain confidence with practice.

This is one of many things one learns over time. Just don't lower your sights. If you aim lower now, it is unlikely you will raise your expectations later. You and your students deserve better.

Date: Mon, 19 Jan 1998
From: Marty Sprinzl

I teach physics to high school sophomores using the modeling method. Although we don't do trig., *we still resolve vectors geometrically*. In Unit 4 and 5, when dealing with objects on ramps and the like, we attack these qualitatively. Objects with a net force of zero must have a constant velocity. For Unit 6, I give students the initial horizontal and vertical components for velocity for which they apply constant velocity or constant acceleration models.

I hope this helps. Our school also has a weak mathematics department. By having sophomores take physics we hope to bridge the algebra gap between freshman and junior years.

Date: Tue, 20 Jan 1998
From: Jon Fishwild <fishj@INXPRESS.NET>
>Question - how does one teach unit 4 without trig? How does one teach unit 5?
Hi G,

I have a lower math ability physics course and I didn't even try to use trig to solve problems. *I just had them do everything graphically by drawing scale force diagrams.* Interesting note, though, many of those kids did poorly on the test--I was very disappointed. Some of the kids who had a trig background decided to use the trig on their own--most of them did OK.

I don't know what I'm going to do next year with this course. I might go the full route and teach the trig so that they are functional, but I'll have to wait and see when the time comes. The point is, it CAN be done without trig.

[Jane's note: read his update below, posted a year later.]

Date: Wed, 21 Jan 1998
From: Larry Dukerich <dukerich@ASU.EDU>

G wrote on the 19th regarding trying to force physics with trig down the throats of his kids. Several Modelers replied and gave good advice; I especially like Rich McNamara's insight. I'm adding my two cents because G asked specifically to hear from the "big wigs". In this matter, however, I feel like a "little wig". My school district adopted a new math curriculum a few years ago in which students spent more time analyzing consequences of the solutions to their problems than in developing facility in solving them. The result has been that the kids have a rotten algebra foundation. They cannot do "math with letters", typically subtracting to undo a factor, and dividing to undo an addition.

Things came to a screeching halt in my classes when we came to Unit 4 simply because students couldn't write F*cos theta to describe a component of a force without first chanting "soh-cah-toa". I spent lots of time encouraging and prodding, and not everyone gets it, but most of my students now tell me how glad they are to have gone through it. They tell me that *they're really learning math now in physics class - perhaps because there is some meaningful context.*

I've concluded that I've got to teach math too, and stop complaining about their weak background because that only makes them feel more inadequate. Some content will have to be sacrificed, but I guess if they start feeling successful, then it's worth it. Besides, the State has recently decreed that in order to graduate from an AZ university, students will have to pass pre-calculus + another math class beyond. So I figure that we're the best chance many of these kids have.

I would say that this is a subject that should be discussed in the first few days of the 2nd summer workshop. Good luck.

Date: Mon, 26 Jan 1998

From: Jerel Welker <jwelker@LPS.ORG> [Jerel teaches math. JJ]

After spending the first semester teaching a block of precal/physics with one of the modeling instructors, *I would highly recommend that you go in search of support from an administrator and a frustrated math teacher. Many math teachers are frustrated by the frequent student response "But, when are we ever going to use ...."* I have yet to hear that question this year! It has been exciting to see students take the mathematics and use it, stretch it, and ask probing questions. Yes, it is more difficult than sitting down and working through the book chapter by chapter. We frequently jump around in the math textbook, but concepts are continually reinforced because we come back to them when the physics warrant. Many math texts are weak at reinforcement.

Secondly, you have a second resource to assist you in teaching such things as trig which can be simplified to nothing more than the poor math student's worst enemy, "the fraction." Perhaps there is a math teacher who is working furiously to teach some of the same concepts because the students just don't get it in their class either. Physics has an advantage in that it is "hands-on" and visual, something that many math classrooms lack. Why not discuss the issue with your math chair or principal?

Date: Mon, 26 Jan 1998

From: Patty Blanton <blantonp@APPSTATE.EDU>

Jon Fishwild wrote:

> [some students] simply have a conceptual problem with the idea of breaking a
> vector into components (just seeing AND BELIEVING that a vector
> can point up and to the right at the same time and that you can actually
> measure how much up and how much to the right it acts).

There has been a great deal of discussion on this point, and Jon's comment about kids being able to "see" components reminded me of a demo I have used that seemed to help. Set a toy truck on an incline. Attach a string to the truck and pass over a pulley at the end of the incline. Tie a weight hanger to the string and add weights until the truck doesn't move up or down the incline. Attach a string to the middle of the truck so that you can pull up on the truck with a force perpendicular to the plane. Pass that string over a pulley mounted on a stand and adjust the height of the pulley until the string is perpendicular to the plane. Attach weights to the weight hanger on the end of that string until the truck is just barely lifted off the plane. Measure the angle of the plane and the weight of the car. With the weights on the weight hangers applying forces perpendicular and parallel to the plane that are equal to the components of the weight of the truck, you should be able to remove the plane and the truck will remain in the same position. Now granted, the parallel force is overcoming both the component of the weight parallel to the plane and friction; the friction is small if the PASCO tracks are used; and this always seems to help students visualize what we mean by "components".

Date: Fri, 27 Feb 1998
From: Paul Gregg Swackhamer <pswackhamer@GLENBROOK.K12.IL.US>

Thanks for the good comments about the difficulty of dealing with regular level students. It has been 14 years since I was in a regular classroom...until this year. It has been an interesting time, and sometimes interesting like a traffic accident :-)!

Looking back at my old notes, it becomes clear how different the "old" course was from what we are trying to do now. But *the students are now at a much lower level than in the past.* Fourteen years ago we only had 35% taking physics at any level. Now we have 75%. And the aggressive commitment to learning by rote by some students is cause for concern.

It is my impression that *the curriculum will need to be designed even more carefully for regular students than for honors to reduce the "activation energy", to borrow a Chem term, without altering the kind of outcome we desire...a coherent argument.*

I believe that there is hope, because I have been working on a wave model with my regular troops with resonating air columns and strings of musical instruments as our "referents" (I bow to Beirut!) Rather than just analyzing these media in terms of boundary value problems, students are able to explain why we get harmonics at certain wave frequencies by keeping track of how wave pulses behave in a computer model. This is not like learning Newtonian particle models; but the arguments they can produce are complete within the context of our simple wave model. And they like the phenomena.

Our challenge is to keep working, it seems, to find out exactly what mechanics deployments can adequately enable them to make the arguments that we desire without becoming overly frustrated and confused. Then maybe we can wean them away from rote learning to satisfying and empowering levels of understanding. I am not there yet in mechanics with these regular students. But I see that they CAN do it in other contexts. My impression is that these students could do nearly as well as honors students on the FCI if they would only be more regularly engaged actively.

My hats off to you all who have been working on this all along with regular students!

Date: Wed, 15 Jul 1998
From: Don Yost <DoYost@AOL.COM>

Always a day late and a dollar short, but for what it's worth: I have many students without trig. I don't require trig from them. *I show them how to solve vectors and trig type problems with scale drawings. They can use protractors and rulers and get acceptable answers. Now what actually happens, is they are drawing along and see other students using trig, and go over and ask what they are doing, and the other students teach them trig.* By the end of the year, all students know how to solve problems using trig, and I never took a minute of class time to do it. If they see a need, and an easier way to do something, they are motivated to learn how to do it; after all, it is only a shortcut.

[Jane's note: Don Yost is an expert modeler in an inner city school. He's a Modeling Workshop leader.]

BELOW ARE 2 UPDATES A YEAR LATER, when the above compilation was posted.

Date: Thu, 21 Jan 1999
From: Jon Fishwild <fishj@inxpress.net>
Subject: Re: modeling w/o trig: a compilation

Jane Jackson wrote:
> 1) How are you using the modeling method w/o trig this year?
> 2) Are you satisfied?
> 3) For those of you who had problems last year, is it better now?

Everything is fine in my honors course; it's the general physics that has posed a problem. Last year I decided that just going slower doesn't cut it. I also tried to do a lot with scaling and just lost kids. So this year, in addition to keeping the pace slower for my general kids, I introduced scaling in Unit IV and we used it to solve a pretty elementary statics problem. Only a small percentage of the test required them to know this; essentially it separated the A's and the B's.

As a result *I've revamped the worksheets dealing with ramps, etc. and do not even give them such problems, other than being able to draw a force diagram.* The only kinds of problems that I expect them to be able to solve are when the forces act along the coordinate axes. I also had to drop Worksheet 3 from Unit V for these kids, as that deals with forces acting at angles too.

I am pleased with how that has worked this year. They seem to have a more solid understanding of the two force models and can apply them fairly consistently to the more simple situations. With the x and y components, they got confused enough to destroy any coherent thinking.

Date: Fri, 22 Jan 1999
From: gheri fouts

This year, I decided to put the trig into the Conceptual Physics and see again what would happen. The Conceptual kids did not retain as much in the way of using trig to problem-solve when it came to the final exam, but along the way, *they picked up the trig and were able to do some of the worksheets* with some satisfaction. They seemed proud of themselves for using something that was new to them, and learning that bit of math as it was applied, rather than as a stand-alone math tool to be used in some course later. But then I have real troopers for conceptual this year. The modeling method is by far the best way to teach physics at any high school level. The worksheets, the labs ... all good stuff.

BELOW ARE TWO RESPONSES TO THE POSTED COMPILATION:

Date: Wed, 3 Feb 1999
From: Tony Nicholson <Tonynic@AOL.COM>

I found the comments on teaching "regular" physics students encouraging. It happens that just today I felt I finally had a breakthrough in my one regular class (so far my only successes have been with my 3 honors classes).....but today was different. For some reason the regular presentations were coherent, without giggling, hemming and hawing and to the point. The students even used words like Net Force, Tension Force, Contact Force, etc without prompting rather than pointing and saying just Force. They took the time to use the nomenclature of naming the kind of force, what it was acting on and finally what was causing it. They even identified which one of Newton's Laws was illustrative of the net Force situation they were presenting. I still had to offer more guidance with questions than I would like but it was a far cry from previous whiteboard sessions. I hope it continues because I was very discouraged and I know now that they can do it.

I think that the mid-term they just had and the fact that I had gone back to traditional presentations for a few weeks because I was discouraged with their previous understanding of the modeling presentation process plus showing them the "Seeing is Believing" video may have had something to do with the change. I also have taken to writing the key words I expect them to use on the corner of the blackboard (much like the Groucho Marx idea of "say the magic word” and win a prize). They actually seemed to enjoy the process and were responsive to questions asked, which never happened for an entire period before.

Date: Thu, 4 Feb 1999
From: Bob Baker <bob.baker@WORLDNET.ATT.NET>
Below is a modeling lab we recently completed to develop sine, cosine, and tangent in *regular* physics. The triangle lab was pre-labbed, run, and presented just like the other modeling labs. Students who had exposure to trigonometry felt the lab was very useful to their understanding. The question, What are the units of sine? , created a great deal of discussion.

Title: RIGHT TRIANGLE LAB.

Purpose: to develop a graphical and mathematical representation of two sides of similar right triangles, one side depending on another side.

Procedure:
1. Draw an angle.
2. Using that angle, create several right triangles.
3. Make a data table of values for one of the following:
   a. side opposite depending on hypotenuse - call this sine (sin)
   b. side adjacent depending on hypotenuse - call this cosine (cos)
   c. side opposite depending on side adjacent - call this tangent (tan)

SOH-CAH-TOA may be used to remember sine, cosine, and tangent.
4. Graph the data table.
5. Linearize the graph.
6. Make a mathematical representation of the graph.

Data evaluation
Conclusion

Follow-up
1. Make a list of the slopes of each person's graph. Note whether they did sine, cosine, or tangent next to their angle.
Examples: sin (390) = 0.63, tan (43.20) = 0.94, cos (11.50) = 0.98
2. Compare the value from each person's graph to the value from your calculator.

ANOTHER POST ON THE SAME SUBJECT, IN 2002:

Date: Fri, 8 Mar 2002
From: Matt Greenwolfe <matt_greenwolfe@CARYACADEMY.ORG>
Subject: Re: Modeling with less mathematical students

I found the discussion with Rex Rice [see the compilation on Physics First, 1999] to be very interesting. At my school all students are required to take physics, but they do it in the junior or senior year, rather than as freshmen. While this is not "Physics First", my *general physics class presents me with some of the same challenges that Rex described - poor algebra skills, low confidence and lack of motivation in science and math, etc. Modeling has really helped to engage this group!*

In addition to *avoiding equations and using more graphical methods to solve problems* as Rex described, I made one modification to units IV and V that has been very successful.

My students have solved all of the problems in those units and more, including problems with forces at angles, on inclines and using the coefficient of friction. But *I made them draw scaled vector diagrams (vector addition diagrams - vectors tail to head, etc) using a ruler and a protractor and stating their scale. Then they *measure* the answers for the unknown forces or the net force off of their diagram and use Newton's second law to calculate acceleration.* With careful technique, they get answers that are close enough to the actual mathematical answers, and *I repeatedly emphasize that I am most interested in the process of solution and their ability to express their understanding in words, graphs and diagrams,* and that the final numerical answer is less important.

The discipline of actually drawing the vectors to scale creates the opportunity for some interesting discussions. Once you know the support force, how long do you draw the frictional force? How is the proportionality between frictional force and support force expressed in the diagram? What if this force or that force or this angle were greater? How would everything else change as a result?

As the semester has progressed, *more and more students have started approaching me and saying, "Do I really have to draw this to scale? Can't I just solve these with trigonometry and the Pythagorean theorem?"* I haven't spent any class time reviewing trigonometry or teaching the students how to solve problems with trig. The students who aren't ready to be more mathematical are still using their rulers and protractors, drawing the diagrams to scale and measuring their answers. The ones who are ready simply start applying what they've learned in their math class with very little additional help needed from me. In fact, *they're highly motivated to use the trig because it saves them time and effort.* Everyone gets to concentrate on the physics primarily without getting the "deer in the headlights" look from the math.

I've been so pleased with the way the scaled diagrams are helping them understand the concepts, that I have started teaching this way in my <u>advanced classes</u> as well. The use of trig suggests itself naturally once the diagram and perhaps a few construction lines are drawn. After a while, I simply have to note that what they are already doing has another name called "finding the components of a vector," which basically turns all the diagrams into rectangles. Rather than a brief introduction to the graphical process of vector addition, followed by introducing components, *the approach helps them understand **why** the method of components works.*

I introduce the scaled vector diagrams after the weight vs. mass lab with a lab on the force table in which the students proposed various different ideas for combining forces pulling at different angles, discussed them with white boards and tested them to see which ones worked. Only the method of drawing all the vectors tail to head was successful. If there was any gap left in the diagram, the central ring would accelerate. I continually refer back to this when students want to simply add or subtract things algebraically.

I wrote a few pages on this for the North Carolina State physics curriculum. The paper demonstrates graphical solutions to a variety of problems and then discusses the transition to trigonometry and components. I would be happy to e-mail that out to anyone who wants a copy.