Simple Harmonic Oscillator: factors that affect the period (awesome approach & results!)

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We just finished investigating the factors that affect the period of oscillation for a mass-on-a-spring and a colleague thought some of the listserv members might be interested in our approach and results.

Discussion provided that the period might depend upon the initial amplitude, the mass on the spring and the spring's properties (i.e., the spring constant). So we decided that each group would use a different spring. We create springs with different "k" values by cutting different lengths from old wave-demo springs as well as using "store-bought" springs, so we have about 15 different springs from which to choose. Each group used one spring and tested the effect of amplitude on the period and mass on the period. But first each group had to determine the value of the spring constant-- a good review from weeks ago! We measured periods with stopwatches instead of motion detectors because I had asked them to make some predictions about the graphs of position-time and velocity-time that I didn't want revealed just yet.

Whiteboarding revealed that amplitude does not affect the period. It also revealed that the square of the period was proportional to the mass attached to the spring when "k" is held constant. The units for the proportionality constant (sec2/kg) were noted to be the same as the units of "1/k", but the slopes did not have the same value as 1/k. Apparently, there is more to the slope than meets the eye!

Each group had agreed to use a 500-g mass trial so that we could share period-"k" data from all of the groups for a "constant mass" trial. With 8 groups in one class and 9 in another, we felt we would have a good spread of data. The graph of period-"k" was "tricky" to find a linear form fit. But a graph of T2 as a function of "1/k" did the trick nicely with a slope of 19.6 kg. Obviously, the slope contains a mass unit but is not the mass on the spring.

In the spirit of combining effects (as Chemistry people do with the combined gas laws), after observing that the square of the period is directly related to the mass and inversely related to the spring constant, it was suggested that we plot T2 as a function of m/k. And so we did. The resulting function:

$$T2 = 39.6 m/k$$

The slope of this last graph is unitless, implying that the constant 39.6 may have special significance. But it isn't immediately obvious... none of "us" recognized 39.6 as being special. But the question was to investigate the factors that affect period, so we re-wrote the function

$$T= 6.29 sqrt(m/k).$$

Still no bells. Oh, well...

We went on to view the motion using a motion detector. As soon as the sinusoidal shape we predicted was confirmed on the screen, multiple hands shot up around the room in each class period. "Could the 6.29 have units of 'rad/cycle'?" "Why would you think that?" "Well, one period is the time for one cycle; but one period is also 2 pi radians in math class. And 6.29 is almost exactly 2 pi radians".

Such excitement, such phun..... Some days, teaching is just AWESOME. It was a great day for me and I just wanted to share.