COMPILATION: Unit 4 - Inventing Vector Addition

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I got my students to invent vector addition - with a little Socratic guidance!

I tried something a little different with my introduction to vector addition of forces. Each lab group had a force table, and I instructed them to put 60g at 0 degrees and calculate the weight (0.59N). I then asked them how we might represent that force on graph paper. All the classes eventually came up with the idea of drawing a line with an arrow, though at first they usually wanted to draw a weight vs. mass graph like they had just done in the last lab. I then asked how long the arrow should be and they eventually came up with a scale of 10 squares represents 1N. I then had them put 80g at 90 degrees, get the weight (0.78N) and represent that with an arrow on the graph paper. At this point they drew them tail to tail because that is what they have been doing for force maps.

Then with no instruction on vector addition of any sort, I just asked them to devise a method for determining the angle and magnitude of the third force that would exactly cancel these two. They had to predict using the graph paper, draw it on their white board, then test it on the force table. Some of them went ahead and found the answer experimentally by trial and error anyway, but I told them they had to come up with a way to determine the force on paper *before* they tested it, and that sent them back to working on the puzzle.

They came up with all sorts of ideas. The most common were to just scalar-add the forces, scalar-average the forces, or subtract the forces in order to find the magnitude of the third force. The most common answer for the angle was that it would be 225 degrees as the two forces would combine to pull at 45 degrees and so the third force had to pull 180 degrees opposite of that. In each class, there was at least one group who knew in advance that this was wrong because one of the forces was greater, so the angle had to be greater than 225 degrees in order to pull more against the stronger force.

I didn't know if they would get any farther than this on their own, however, so I was a little nervous trying this out. But in each class, there was at least one group who had the idea of using the Pythagorean theorem to find the magnitude of the third force. I had this group present last. Then I asked them since they were using the Pythagorean theorem, there had to be a right triangle. Where was the right triangle? Could they make a right triangle with their three forces? It took some trial and error for them to figure out how it all fit together. They kept wanting to keep them tail to tail, but flip the directions. I had to keep repeating that they could not change the direction the force pulled, but that they could move the vector around to make it fit into a triangle. Once they had the triangle, I then asked what would happen if the force were too little or too great, or if the angle were wrong. It was obvious that the figure would then fail to be closed and the forces would not cancel.

In my first period class, one group treated the two forces like a slope. 0.59N in the x direction and 0.78N in the y direction gave them a slope of about 1.32. They drew this line and measured the

angle. In the same class, I had another group actually draw the other two sides of the rectangle starting with the initial two forces and then use the diagonal to find the resultant force. They knew the third force would have to be opposite the first one, but didn't know how to measure the angle. Combining the two groups, they basically had developed the whole idea themselves! The idea of putting the vectors tail to head and drawing a triangle was then just a neater way of presenting what they had already figured out.

I think by doing it this way, they will learn more about which answers (like scalar addition or just averaging the angle) are incorrect and therefore be more confident in the correct vector addition procedure when they start seeing more difficult problems. This is my first year of modeling, and I am really enjoying it. I would have never guessed that my students would have been able to do so much themselves. It is so much more exciting this way than just presenting them with a method of vector addition and then applying it. I am planning to have them keep using the ruler and protractor all the way through units 4 and 5 and then introduce the trigonometry at the end.