COMPILATION: unit 7: Force versus stretch (Hooke's law)

Date: Thu, 6 Mar 2003
From: Rob Spencer <rspencer@KIVA.NET>
Subject: Hooke's Law question

My students are going to argue between a quadratic fit and a linear fit for the graphed data on force vs. stretch tomorrow. The argument hinges on a comparison between the R^2 values that Excel gives for correlation. An R^2 value of 1 means that the best-fit is an exact fit. Their correlations are going to be 0.997 (quadratic) vs. 0.995 (linear) for example. Any suggestions? Development of important energy considerations depends on F=kx.

Date: Fri, 7 Mar 2003
From: Rob Spencer <rspencer@KIVA.NET>

In response to my own musings: "...and I also wondered about the horizontal intercept which says that when the force is zero, the stretch is negative...is the displacement from that horizontal intercept over to the origin a measurement of how much the spring stretches under its own weight?"

This assessment does not make sense because all of the springs had the coils tightly bunched with no load.

Date: Sat, 8 Mar 2003
From: Art Woodruff <woodruff_a@YAHOO.COM>

Quadratic can't be correct because if you keep going with the data, eventually a larger stretch requires less force, not more, as the parabola comes back down to 0.

Date: Sat, 8 Mar 2003
From: Jean OOSTENS <joostens@CAMPBELLSVILLE.EDU>

The correlation coefficient is convenient for social sciences, where there is a considerable scatter of the data. For example a correlation of 80 % between the number of pack of cigarettes smoked per day and the number of lung cancer cases would be a smoking gun to ban the consumption of tobacco. The same percentage in physics would indicate a very poor fit of the data to a straight line.

In other words, the difference between 0.999 and 0.997 would be quite sufficient to tell which fit is best. An alternate argument has to do with the implication of an extrapolation: would the fit be meaningful outside the interval where the data were taken?

In your case, stretching the spring a little farther would tell whether the second degree term is justified (it might be the case for a spring that has gone over its normal limit of use).

Another argument is: "Keep it simple". If proportionality works fine, do not seek additional adjustable coefficients just to reach a better correlation coefficient. Remember that if you have N points, a N-1 degree polynomial will fit perfectly!

Date: Sat, 8 Mar 2003
From: Matt Greenwolfe <matt_greenwolfe@CARYACADEMY.ORG>

I get a lot of mileage out of asking the students about the x and y intercepts of their curve fits and the limiting cases as x approaches zero or goes to infinity or negative infinity, etc. In this case, the segment of your quadratic fit that passes through your data is probably not curved very much, so that the vertex will lead to a peak at a positive value of x, or a minimum at a negative value of x, depending on which way the parabola curves. But the vertex is probably nowhere near 0,0.

In the case of the vertex at a positive value of x, ask them to scroll to the right on their graph (and perhaps up) until they find the vertex. This is easy in Graphical Analysis, and Excel can surely do the same thing in some manner. Then ask what happens at values of x past the vertex. They should conclude that the force continuing to decrease as the spring stretches further does not make sense. If it's physically possible to test that without ruining the spring, so much the better.

If the issue of permanent deformation of the spring comes up at this point, you can argue that the parabola doesn't model that well either.

In case of the peak at a negative value of x, you will have to talk them through a negative x being a compression, if the spring could compress, and leading to a force in the opposite direction; then proceed as in the positive case to discuss the decrease in the amount of force after the vertex.

In the case of the vertex at or near 0,0 you have a compression leading to a force in the same direction as a stretch, which does not make sense. I then ask them which curve fit best fits their data and also makes sense in the limiting cases.

Date: Sun, 9 Mar 2003 1

From: Rob Spencer <rspencer@KIVA.NET>

Thanks so much for the responses w.r.t. my initial inquiry. You verified what I suspected in my 2nd post, that weights not causing any stretch should not be included in a linear fit. I would like to share some final comments on some of the topics you touched upon.

<u>Quadratic fit:</u> within the range of weights tested, the quadratic fit works nicely as evidenced by the high correlation value. However, so does the linear fit. The students have already discussed ways of making a clear-cut determination - one of which includes expanding the range of added weights and also filling in-between previously tested values. Both results being equal, I will guide them to use the linear fit. Of course, they are going to run into the elastic limit. That's all right because then both models should break down if they are good up to then - and the least expensive springs purchased were about $0.32 apiece and can be sacrificed for the good of science.

<u>Linear fit:</u> I do believe that all springs gave very nice data for students to agree that a linear fit is appropriate - onward to energy topics!

<u>Non-linear force:</u> this idea came up from an off-list memo from a very astute modeler. The idea being that maybe the springs that I purchased might more appropriately be fit with higher order terms (than just being a linear restoring force). Since I have no data to go on other than what students collect and (admittedly what I checked personally on a Sat. night), I agree that this is a possibility. This modeler suggested looking up such a possibility in higher order physics books. Tonight, I found my "Classical Dynamics" book by Marion. What I found was very interesting w.r.t. the type of springs that I had (very symmetrical data). Marion suggests that each real case of a restoring force should be treated on a case by case basis and that "In many physical situations, the deviation of the force from linearity is symmetrical about the equilibrium position..." which would cause the relationship between the force and stretch to have a linear term (Hooke's Law) and then a cubic term. (nice suggestion Marc!)

This would suggest that the quadratic fit is in fact artificial...but still worth investigation by the students.