

COMPILATION: Unit 7 - KE of a rolling ball

Date: Tue, 5 Feb 2002

From: Gheri Fouts <gfouts@pixi.com>

Subject: KE of a rolling ball

Perhaps you all can help me with this strange outcome.

We are in Conservation of Energy and I wanted to have a sphere roll down an incline. Since a big carnival event just finished last weekend, I pulled out the wooden roller coaster tracks and the students decided to find a relationship between velocity and height of track. They could not use the Vernier photogate attached to a computer since this school does not have computers, but they used a CPU photogate timer that plugs into the wall via a small transformer. According to their figuring, the slope of the V^2 vs height line should be $2g$.

I went around to check their setups and calculated the velocity at the bottom of some of their setups just based on the height of the top of the track minus the height of the bottom of the track, and I saw a 13% error between the calculated V and their actual V found by diameter of ball/time from the timer. After they left, I tried it myself and still got a 13% error. The track has a hill in the middle of it so I figured there must be friction unfortunately involved. I tried it again with the photogate at the bottom of the first hill and I still got this 13% error. So, I pulled out the Pasco ramp and put on a Pasco car with an index card mounted on top. I ran that sucker through the photogate, and I still got a 13% error. So, what is going on? Could the electricity be a slightly different frequency and therefore the time is not correct which throws the whole thing off? I am using $KE_{max} = PE_{max}$ or $v^2 = 2gh$, solving for v and comparing it to the measured v found by diameter of ball/time measured on timer. I can't believe it is friction.

Date: Wed, 6 Feb 2002

From: Richard McNamara <richmcn@EARTHLINK.NET>

Gheri Fouts wrote:

<<... I saw a 13% error between the calculated V and their actual V found by diameter of ball/time from the timer. After they left, I tried it myself and still got a 13% error. >>

As the ball leaves the track, are all parts of the sphere moving at the same speed? Since it's rolling, the answer is no. The part of the ball in contact with the track (provided the ball isn't sliding) is ZERO. The top of the ball has a speed of $2x$ the center of the ball. This combination of

linear motion along the track combined with the rotation of the ball results in a slightly different amount of kinetic energy being present in the ball. It's still $\frac{1}{2}mv^2$, but there is also rotational kinetic energy = $\frac{1}{2} \times \text{moment of inertia} \times \text{rotational speed}^2$.

The moment of inertia for a solid sphere rotating about its center is given by:

$$I = \frac{2}{5}MR^2.$$

The rotational velocity is found by the formula:

$$\text{rotational speed} = \text{linear velocity} / \text{radius of rotation}.$$

So

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \left\{ \frac{2}{5}mr^2 \right\} \left(\frac{v}{r} \right)^2$$

$$gh = \frac{1}{2}v^2 + \frac{2}{10}v^2 \text{ or } \frac{7}{10}v^2$$

so

$$\frac{10}{7}gh = v^2.$$

If we compared $\frac{10}{7}g$ to $2g$, you get a ratio of $\frac{1.4}{2}$ or around 70%.

This assumes that the ball doesn't slide at all. If there's some sliding, you will get a larger final linear velocity.

I do this lab with my AP class (when I have one) and we re-examine the data when we get to rotational motion to see why the slope of the original lab was less than $2g$ and usually get better results. I hope that helps.

Date: Thu, 7 Feb 2002

From: Matt Greenwolfe <matt_greenwolfe@CARYACADEMY.ORG>

I've also done this demonstration with my Advanced Physics classes. Last year, I had a group of students who just wouldn't be satisfied that we had accounted for everything after we included the rotational motion as per Rich's posting. We noticed that the ball rolled on a "V" shaped track and so the ball did not contact the track at its diameter, but at two points much closer to the center of the ball. Noting that our track's groove made a 90 degree angle, it was just a matter of geometry to calculate the distance of the point of contact from the center. When calculating the moment of inertia of the ball, still use the ball's radius, but when relating angular velocity to linear velocity through $v = \omega \times r$, use the new distance. On our track, this makes a noticeable improvement in accuracy. We were within a few centimeters of the calculated height and the students were very impressed.
