COMPILATION: Unit 9 - Rain on wagon problem in AP-B exam

Date: Thu, 26 Apr 2001
From: John Barrere <forcejb@YAHOO.COM>
Subject: 1998 APB exam

During a review session, one of my rather bright students argued that question 4 in the 1998 APB MC section could be answered "decrease on the basis of conservation of mechanical energy" just as well as using conservation of momentum. This is the question with vertical rain falling into a wagon rolling w/o friction. His argument was since energy is constant (in the absence of any force component in the direction of motion), if the mass of the wagon increases due to accumulated rainfall, then velocity must decrease (to keep energy constant). The 'correct' answer is momentum-based, but I can't find a reason to shoot down my student's answer. Am I just unusually dense today??? Be gentle - it's been a tough week.

Date: Thu, 26 Apr 2001
From: "Richard J. McNamara" <richmcn@EARTHLINK.NET>

I don't think you can use an energy argument because strictly speaking, the energy is not constant. Each raindrop does have some kinetic energy before it strikes the wagon's surface. It really complicates the heck out of things trying to figure how the system's energy changes as a result of each drop's interaction with the wagon, but it does preclude saying the system's energy is constant.

Date: Fri, 27 Apr 2001
From: Don Yost <DoYost@AOL.COM>

Rain has kinetic energy which is transformed mostly into heat which is dissipated into the surroundings. Mechanical energy is not conserved in this case. Mechanical energy is only conserved for "springy" interactions.

Date: Fri, 27 Apr 2001
From: Thorn Green <viridian_1138@YAHOO.COM>

I'm not sure about the reply to the question about rain falling into the wagon. If the rain falls in a direction orthogonal to the direction of travel of the wagon, then it seems that the impulse imparted by each individual raindrop will act strictly against the normal force between the wagon wheels and the ground.

If the wagon truly rolls in a frictionless manner, then the increase in normal force should not cause a change in the kinetic energy of the vehicle.

Date: Fri, 27 Apr 2001
From: Bob Sciamanda <trebor@VELOCITY.NET>

Each collision of a raindrop with the wagon is a totally inelastic collision, which must dissipate kinetic energy into other forms in order to conserve momentum. To see this, analyze one such collision.

Date: Fri, 27 Apr 2001
From: "Richard J. McNamara" <richmcn@EARTHLINK.NET>

Sounded like a pretty 'expert' type answer to me. On the other hand, if you consider the impulse issue:

If the raindrop is traveling straight downward like you said, the component of its velocity in the direction the wagon is traveling will be zero. After it hits the wagon it must end up traveling in the direction of the wagon's motion with the same speed as the wagon. What is it that accelerates the raindrop in the direction of the wagon's motion? The interaction between the wagon and the rain drop. The wagon pushes the raindrop 'forward' so the raindrop pushes 'backward' on the wagon. The net force on the drop and the wagon ends up being zero, but the cart slows down because of the pushes from the raindrop. The net force is what says the system's total momentum in the direction of travel remains the same, but since the mass increases, the wagon slows down.

On the issue of frictionless motion and a changing normal force, that's another one of the reasons that the energy argument won't work. The scalar nature of energy and energy transfer is what keeps that from being an easily solvable problem.

Date: Sat, 28 Apr 2001
From: Jason B Lonon <j_lonon@LYCOS.COM>
Subject: raindrops keep falling on my cart

I was thinking about the rain falling on the wagon, and something about the comment below struck me as strange.

<<
If the raindrop is traveling straight downward like you said, the component of its velocity in the direction the wagon is traveling will be zero. After it hits the wagon it must end up traveling in the direction of the wagon's motion with the same speed as the wagon. What is it that accelerates the raindrop in the direction of the wagon's motion? The interaction between the wagon and the rain drop. The wagon pushes the raindrop 'forward' so the raindrop pushes 'backward' on the wagon.
>>

Here's the catch. If the rain is _really_ falling vertically _with respect_ to a moving cart, then each drop must have exactly the same horizontal component of velocity as the moving cart. In this case, there is no impulse delivered in the horizontal direction, since the drop is never accelerated in that direction. Does the cart slow down now?

Date: Sat, 28 Apr 2001

From: Bob Sciamanda <trebor@VELOCITY.NET>

Let's divide the phenomenon into two processes:

1) First the vertically falling drop is brought to rest by its collision with the earth/wagon system. In the limit as the earth mass (M) is taken to be "infinity" its recoil velocity (V) becomes infinitesimally small. But in this limit the product of the infinite earth mass and its infinitesimal recoil velocity (MV) is finite and just equal to the original vertical momentum of the raindrop. Now the kinetic energy imparted to the earth is that finite product MV multiplied by another infinitesimal V (and an irrelevant factor of 1/2). So the kinetic energy acquired by the earth is infinitesimally small (zero). Thus the earth takes up all of the original, vertical momentum of the drop, but the earth acquires none of the original kinetic energy of the drop - this energy is completely dissipated as sound, "heat", etc. (If there is no mechanism for such dissipation, the collision will be elastic, and the falling object will bounce.)

This part of the problem is no different than dropping a lump of putty onto the earth. Both involve a totally inelastic collision, with M/m => infinity, which conserves momentum and dissipates kinetic energy.

2) Now the drop becomes attached to the wagon floor and is accelerated up to a final, common, horizontal velocity. This is another totally inelastic collision, this one involving horizontal motion, in which momentum is conserved and kinetic energy is dissipated.

Note that, depending on the speeds and masses involved, the amount of kinetic energy dissipated in process (1) may be more, less, or the same as the amount dissipated in process (2).

Venturing farther afield, note that if you run a movie of a totally inelastic collision backward, you see a "jet propulsion" (or "gun firing a bullet") process in which momentum is conserved and non-kinetic energy is transformed into kinetic energy, obeying all of Newton's laws. The examples chosen to view may or may not violate the second law of thermodynamics. (Think of a gun using gunpowder vs a pop gun using a latching spring.)

Date: Sun, 29 Apr 2001

From: Vonnie Hicks <VMHicks@AOL.COM>

Concerning your question about water dropping into a moving cart, and to enlarge on Bob's description of the raindrop colliding inelastically in the horizontal direction with the cart, there must be an Ediss due to the working which accelerates the raindrop horizontally. Although this is not the usual friction situation as one might expect with the dropping of, say, sand, there would be contribution to the chaotic motion of the water in the cart making the required amount of energy no longer available for the horizontal motion of the center of mass. Using water instead of some particulate solid certainly adds a new level of possible confusion to this problem!

Date: April 29, 2001
From: Thorn Green <viridian_1138@YAHOO.COM>
Hi Rich,
I was so interested by the problem of whether the wagon would conserve kinetic energy that I decided to try to derive what the solution should be. I don't have a copy of the original question, so please excuse me if I set up the problem incorrectly.

I think the wagon wheels are a confounding factor, so instead of a wagon I am going to assume that the problem concerns a frictionless block with a large hole at the top to accept water.

I assume that the block slides on the X-Axis from negative to positive with a velocity v0. I also assume that the rain falls from positive Y infinity to negative Y infinity in a straight line.

I start with Newton's equation:

$$\sum_i F_i = \frac{dP}{dt}$$

Where the Fi are the force vectors on the block, and P is the momentum vector of the block. In my interpretation the Fi consist of three forces: the force imparted by the falling rain upon the wagon at a particular instant which I call F0, the weight of the block which I call F1, and the normal force of the ground onto the block which I call F2.

Hence the equation becomes:

$$F0 + F1 + F2 = \frac{dP}{dt}$$

The derivative of P breaks into the familiar terms:

$$F0 + F1 + F2 = v * \frac{dm}{dt} + m * \frac{dv}{dt}$$

where m is the mass of the block, the v is the velocity vector of the block. Because of the influence of the accumulating rainwater, the time derivative of the mass is non-zero. If one draws a free body diagram corresponding to this equation it seems clear that F0, F1, and F2 are all parallel to the Y-Axis. Meanwhile, v and the time derivative of v are parallel to the X-Axis. Hence, I believe we can do the following separation of equations:

$$F0 + F1 + F2 = 0$$

$$v * \frac{dm}{dt} + m * \frac{dv}{dt} = 0$$

The top equation essentially says that the normal force cancels the weight plus the force of the rain. The bottom equation describes how the change in mass affects the motion of the block. This can be changed into the following:

$$v * \frac{dm}{dt} = - m * \frac{dv}{dt}$$

With some more manipulations:

$$\frac{\frac{dv}{dt}}{\frac{dm}{dt}} = - \frac{v}{m}$$

And with some more manipulations:

$$\frac{dv}{} = - \frac{v}{}$$

$$\frac{\ }{dm} \quad \frac{\ }{m}$$

Relaxing the idea that v is a vector, and taking v in one component:

$$\frac{1}{v} dv = - \frac{1}{m} dm$$

Then integrating both sides:

$$\ln(vf) - \ln(vi) = \ln(mi) - \ln(mf)$$

Where mi is initial mass, mf is final mass, vi is initial velocity, and vf is final velocity. Then rewriting in terms of the final velocity:

$$\ln(vf) = \ln(mi) - \ln(mf) + \ln(vi)$$

And some more manipulations yet:

$$\ln(vf) = \ln\left(\frac{mi * vi}{mf} \right)$$

And then this pops out:

$$vf = \frac{mi * vi}{mf}$$

This yields exactly what one would expect from a conservation of momentum argument.

The initial kinetic energy of the block is thus:

$$0.5 * mi * vi^2$$

And the final energy turns out to be:

$$2 \qquad mi * vi \quad 2$$

$$0.5 * mf * vf = 0.5 * mf * \left(\frac{\text{------------}}{mf} \right)$$

$$= 0.5 * mi^{2} * vi^{2} * mf^{-1}$$

So, unfortunately for the student, it seems that energy is not conserved in this model.

Note that in this model momentum conservation is maintained without any external force from the raindrops acting in the block's direction of motion. That is to say, the raindrop does not push "backward" on the block. Instead, the velocity slows strictly due to the accumulation of mass.

Anyway, that's just my two cents on the matter.

Date: Mon, 30 Apr 2001
From: John Barrere <forcejb@YAHOO.COM>

Thorn - Nice work!! But, what happens to the "lost" energy? It must be dissipated (as temp increase in wagon/water?), but that begs the question as to whether more or less could be dissipated given different circumstances. See, I'm "cursed" with Swackhamer's questions re: energy: Where does it come from, where does it go, what does it do? I could see the GPE showing up as "heat" but what about the other "lost" energy???

Date: Mon, 30 Apr 2001
From: "Lou C. Turner"

I liked everything about Thorn Green's mathematical analysis of the rain and the wagon problem except the following sentences.

<< … the raindrop does not push "backward" on the block. Instead, the velocity slows strictly due to the accumulation of mass. >>

A raindrop does not move in the direction the cart is moving unless a force makes it do so. The force is provided by either the friction force the cart exerts on the drop or by a collision with the back end of the cart. In either case the raindrop exerts a force on the cart that slows it down.

Date: Mon, 30 Apr 2001
From: Joseph Vanderway <jvanderway@CSUN.EDU>

I think this is a bit of a dead horse. Mechanical Energy is clearly dissipated in the collision between the water and the cart. I post (it's my $1.50 towards the discussion) to warn against a possible error in these mass transfer problems:

From the middle of Thorn's argument...

<<

$F_0 + F_1 + F_2 = dP/dt$

The derivative of P breaks into the familiar terms:

$F_0 + F_1 + F_2 = v * dm/dt + m * dv/dt$

>>

in this case, dp/dt only equals $v*dm/dt + m*dv/dt$ because the initial horizontal momentum of the water is zero. The problem is that there are several particles contributing to the momentum, so the momentum of the system can not be considered as $p=mv$ for a single particle.

For more on these types of mass transfer problems, please see "An Introduction to Mechanics" by Kleppner & Kolenkow Chapter 3 section 5. To avoid any confusion, K & K recommend the following strategy (as interpreted by me):

1. Clearly define the system including a small mass element that will be transferred in a delta t. (ex. cart + water in cart + one drop that will land in the cart during a time delta t)

2. Write an expression for the total momentum of the system before the mass is transferred. (pi)

3. Write an expression for the total momentum of the system after the mass is transferred. (pf) Note: this usually involves a (v + delta v) to describe the velocity after the transfer.

4. Subtract pi from pf to get delta p.

5. Divide delta p by a delta t and take the limit as delta t becomes zero. This usually converts any deltas to differentials (delta v becomes dv, etc.) Any terms with any differentials alone at the end - not part of derivatives like dv/dt become zero. This gives you an expression for dp/dt.

6. Set sum of forces acting on the system equal to dp/dt and solve for what you are looking for.

Since the sum of horizontal forces in the wagon problem is zero, and the small mass element (water drop) has a horizontal velocity of zero, the end result is the same: Vf = mi*vi/mf.

Date: Mon, 30 Apr 2001

From: Bob Baker <bob.baker@WORLDNET.ATT.NET>

Suppose in the wagon problem that instead of raindrops landing in the wagon, we drop a super ball with very high elasticity and high coefficient of friction. The ball strikes the bottom of the wagon and bounces back up out of the wagon regaining its original height and potential energy. The ball is now spinning from the frictional contact with the wagon and has gained rotational energy. The wagon has less kinetic energy because of the impulse it gave to spin the ball. I would guess that the raindrops also tend to spin when they hit the wagon.

Date: Fri, 4 May 2001

From: Thorn Green <viridian_1138@YAHOO.COM>

Subject: Re: Wagon Question Responses

Once again, I'm way behind on getting things accomplished. Nevertheless, here are my responses to some of the posts on Apr. 30.

\> Date: Mon, 30 Apr 2001

\> From: Joseph Vanderway jvanderway@CSUN.EDU

<<The derivative of P breaks into the familiar terms:

F0 + F1 + F2 = v * dm/dt + m * dv/dt

in this case, dp/dt only equals v*dm/dt + m*dv/dt because the initial horizontal momentum of the water is zero. The problem is that there are several particles contributing to the momentum, so the momentum of the system can not be considered as p=mv for a single particle.

\>>

According to my physics text, P = m * v is a definition. Any particle at any instant in time has a momentum equal to m * v. Since the expression for P is a definition, taking the derivative of both sides should yield an equation that is always true. So I don't agree with the statement about dP / dt being possibly in error.

In the derivation I presented, the wagon plus the sum of all water it accumulated at time "t" is being modeled as a single system. The rain, until it is accumulated, would thus be an external input to the system. So if the rain exerted a horizontal force due to collision, this force would contribute as a horizontal component of F0 (as defined in the original E-Mail post). It seems that an added horizontal contribution in F0 would produce the same results as adding an extra horizontal term to dP / dt.

I think that modeling the rain plus the wagon as a single closed system produces a perfectly valid (and equivalent) alternate solution. However, that is not how I chose to model the problem. And I am not sure that these two alternate methods for solving the problem are easily mixed

[Lou Turner wrote:
<<
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>>

When I say that the rain does not "push back" on the cart, what I mean is that the horizontal component of the force called F0 (in the original E-Mail post) is zero. If one looks carefully at Newton's equation, there ARE cases where a particle with no forces acting on it can change velocity. A change in particle mass over time results in a change of velocity independent of any external force.

<< … what happens to the "lost" energy? It must be dissipated (as temp increase in wagon/water?), but that begs the question as to whether more or less could be dissipated given different circumstances. See, I'm "cursed" with Swackhamer's questions re: energy: Where does it come from, where does it go, what does it do? I could see the GPE showing up as "heat" but what about the other "lost" energy??? John Barrere
>>

It is interesting to note that the cart could regain its original kinetic energy by dumping the water mass in a similar manner to which it was accumulated.

Saying this seems strange, but it appears as if the accumulation of the water leads to the storage of a kind of potential energy. And then this potential can later be turned back to kinetic energy. For instance, consider a rocket sitting on a launch pad. The rocket has the potential to go into space, even though it has a kinetic energy of zero in the reference frame of the launch pad. When the rocket expels some of its mass as exhaust, it then gains that kinetic energy.

Date: Fri, 4 May 2001

From: Joseph Vanderway <jvanderway@CSUN.EDU>

Returning to the horse...

If the system (wagon + water already inside) is considered a particle, then the

dp/dt = m*dv/dt + v*dm/dt

argument will hold. The only problem here is that you set the sum of horizontal forces interacting with the defined system equal to zero. However, if the sum of forces in the horizontal direction acting on the system is zero, then the system will move with a constant velocity - which is not true in this case.

Or to look at it another way...

I think Lou is correct on this one.

If the water has an initial horizontal velocity of zero, and a final horizontal velocity which is non-zero, then there must have been a horizontal force which accelerated the water. The only interaction that can occur to cause this force is between the wagon and the water. By Newton's third law, there must also be a horizontal force on the wagon and thus the sum of horizontal forces on the defined system (wagon and accumulated water at time, t) must not be zero.

I believe this is what Lou is saying here...

<< A raindrop does not move in the direction the cart is moving unless a force makes it do so. The force is provided by either the friction force the cart exerts on the drop or by a collision with the back end of the cart. In either case the raindrop exerts a force on the cart that slows it down. >>

Thorn responds:

<<

When I say that the rain does not "push back" on the cart, what I mean is that the horizontal component of the force called F0 (in the original E-Mail post) is zero. If one looks carefully at Newton's equation, there ARE cases where a particle with no forces acting on it can change velocity. A change in particle mass over time results in a change of velocity independent of any external force.

>>

I confess I do not understand this... How is this possible? How does a "particle" change its mass (outside of relativistic considerations)? Help? Am I missing something?

[Jane's note in Jan. 2003: if you want to answer Joseph Vanderway's question, please cc to me and I'll add your response here. jane.jackson@asu.edu]