COMPILATION: linearizing improves proportional reasoning (Joe Mahler has evidence)

[ed. note: for continuity, see the Sept. 2016 compilation on Linearizing Worksheet of Joe Mahler. Joe posted that important worksheet and other linearizing documents at https://drive.google.com/drive/folders/0B8bJkNnVbHRGTHgwNGZjZGtPN28?usp=sharing AMTA members can download the worksheet & readings in Unit 1 of the mechanics materials. Joe wrote: “Since I have been teaching linearizing the way I currently do, I have seen my students' proportional reasoning greatly increase, and almost every student now gets the proportional reasoning question on the Lawson Test correct.”]

From: David Ennis
Date: Wed, 24 Sep 2014

linearization can help students discriminate between a linear fit and one of the nonlinear fits on noisy data. If it’s needed for that, and especially if it fails at that, the message I make sure the students get is that they need to take greater care with their measurements, and to make sure they have on the order of a 10 to 1 ratio between their largest and smallest settings of the I.V.. If you're consistent about getting all relationships from graphs of their data even in circular motion, as I am, linearization will show them that their graph of velocity vs. m is showing an inverse square root, and not just an inverse. …

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From: Joe Morin
Date: Thu, 25 Sep 2014

I really like the method of linearization, especially with noisy data as David mentioned. Curve fitting often introduces additional polynomial terms that confuse the model. I don't teach the Scientific Reasoning unit as a separate unit, so we do linearization as a technique when we get to models that need it. By then students have a stronger understanding of the linear graphical relationship between two variables.

When we get the resulting equation after linearizing, I ask students to tell me to which graph does it apply. The first knee jerk reaction is that it only applies to the linearized graph. Then we have the discussion about it being the equation of both graphs, and linearization is just the technique for finding it. I don't think it's confusing after that, and my AP students who know how to curve fit find it to be an interesting technique.

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From: Bob Bogenrief
Date: Thu, 25 Sep 2014

I would suggest another reason for linearization that may relate to those who have students with less than desirable math skills. Some students can learn to make physical sense of the slope and intercept of a linear equation when they have not learned how to work with higher level equations. Students who are weak in linear equations can learn from the consistent practice provided in a modeling class. I once had students in a physics class take an advanced chemistry class the following year. The teacher had taught these same students in a chemistry class the year before they took physics. The teacher observed that in the second year course they readily understood density as the slope of a graph of mass versus volume and that they would not have been able to do so in the year prior to the physics class. Credit was given to their experience in physics for their growth in skill and understanding.
It is true that students can easily become confused by the change from, for example, a quadratic graph in x to a linear graph in x^2. This can be overcome by sketching the two graphs side by side and leading students to recognize the difference in the horizontal variables and the resulting change in the form of the graph. Compare the equations by where the parentheses are placed. Quadratic is y = k(x)^2, linear is y = k(x^2). It also helps to emphasize and repeat whenever the occasion presents itself that the change is being made because the linear form makes certain physical meanings easier for the students to grasp.

I personally think that linearization is an ingenious way to help students understand physics mathematically.

From: Michael Tollefson
Date: Sat, 27 Sep 2014
I believe linearization is a key piece in the analysis of the paradigm labs throughout the year. I use it with both my accelerated classes and CP classes. It takes a bit of hand-holding at the beginning of the year, but the students slowly pick it up as the year moves along. The math teachers at my school like the summary of graphical analysis possibilities taken from unit 1 (the one page summary).
It can be hard to tell if a relationship is quadratic versus a higher order without the transformation. All of my students know y = mx + b (they are seniors!) so this is a great starting spot. Manipulating a quadratic curve or hyperbola is a black box to them. When they see a similar shape pop up when doing circular motion, they seem to remember they saw it when they did acceleration and can linearize it. When they examine Ohm's Law they see the similar graphic pattern when they did Newton's Second Law. It is a worthwhile technique that helps everyone better understand curved lines easier.

From: DONALD YOST
Date: Sat, 27 Sep 2014
Linearization. Whether to use or not to use: I think it depends on your goals. If you are interested in the "answer", then use a computer to find the relationship. If you are interested in conceptual understanding, then use linearization.
I think the essence of this problem is mirrored in the process of division. Most students learn the division algorithm in the third grade. In their minds they know how to divide. However, studies have shown (Schoenfield) that these students will probably never learn what division actually is. They know how to divide: so no problem. You have met these students in class. They try a into b, then b into a, and just pick the answer that looks best. Had they learned the concept first, as through CIMM, then learned the algorithm, they would actually understand what the process is.
Linearization may be tedious, but it gives a concrete understanding of what we are doing, rather than just a "magic" answer.

Date: Mon, 29 Sep 2014
From: Joseph Mahler
Subject: Repost about Linearizing
It seems that a discussion about linearizing comes up every year. Below is a post that I posted in 2011 on the subject. I want to say that I have successfully done linearizing with 9-12 graders and I think it can be used to teach some very important skills/understanding.
Since I have been teaching linearizing the way I currently do, I have seen my students’ proportional reasoning greatly increase, and almost every student now gets the proportional reasoning question on the Lawson Test correct.

From: Joseph Mahler
Date: Fri, 16 Sep 2011

I've been following the postings about linearizing for the past week or so and have hesitated to post myself. If I am remembering correctly, this discussion came up last year about the same time and I posted my piece then. However, with the resurfacing of this discussion I felt it important to restate what I really think the importance of linearizing is.

I think the basic flaw that most science/physics teachers make is they assume their students have a decent understanding of proportions. I speak only from my own personal experiences (12 years of teaching a variety of levels of physics), but even students who I teach that are in calculus do not understand proportions. I believe that the real reason to teach linearizing is to help students develop a solid understanding of proportions -- NOT to get equations. (If you are teaching linearizing to simply get the students to find an equation, then I would argue the case for not even wasting your time or the students' time. Simply have the computer do it and the students will have probably the same amount of understanding of what is going on.)

By building a solid understanding of proportions and using linearizing as a teaching tool to do this, later in the year when you are studying acceleration the students will understand what is meant when we say position is proportional to time^2.

To accomplish this, I spend time with the students looking at simple functions (y=2x, y=2x^2, y=2(1/x), etc.). I have them graph each function, and for each function calculate the delta(x) and delta(y) over a variety of ranges. The students then compare the delta(y) to the delta(x) for the various functions. They quickly realize that for y=2x function that as we change x by 2 times as much, 3 times as much, etc. that the change in y is 2 times as much, 3 times as much, etc., and that if we divide out delta(y) column by our delta(x) column, we get a set number. This is when some student blurs out that this number matches the slope of the graph, and the class makes the connection between the two.

From here we establish that if two variables are proportional, then the ratio of the delta(y) to delta(x) is constant, you get a straight line graph, and the proportionality constant is the slope of the graph.

Then by looking at the ratio of delta(y) to delta(x) for the other functions, the students establish that the graphs of the other functions are not linear because the variables plotted are not proportional to each other.

Also the students make observations such as for the y=2x^2 function, as x changes (2 times as much, 3 times as much, etc) y changes by 4 times as much, 9 times as much, etc.

I then ask the students, “if we made a third column that was proportional to delta(y), what would it be?” They quickly say we should square the x values and then subtract them. I have them do that and I ask them “are the two proportional?” They say yes because when we divide the two we get a set number (our proportionality constant).

Once we establish this, I ask them if we were going to make a second graph for our y=2x^2 function and I wanted to have a straight line, what should we do? The students are able to tell
me to plot $y$ vs. $x^2$ because we have established that if a graph is linear it simply means the two variables plotted are proportional and they know $y$ is proportional to $x^2$.

This is the key because the students are figuring out themselves how to "linearize" and they understand why it works. It is not a trick. When I question the students on why the non-linearized graph and linearized graph look different, they quickly point out that the two graphs represent the same function, just the variables plotted are different. Also they point out that the slope of the linearized graph is the proportionality constant we found.

We then as a class then go through and do this for each of the remaining functions.

Doing this I have found the students have a true understanding of linearizing, and when they say to me “the position is proportional to time$^2$”, I know they really understand what they are saying.

I would again argue that if you go through the linearizing process and the students don't recognize that the non-linearized graph and linearized graph are really showing the same information, then they don't understand what they are doing. Also if you are simply having them memorize shapes and know what to do with what shape, then you are not really accomplishing anything. Just have the computer do it. I'm sorry if I offend anyone, but that is my two cents.

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From: Joseph Mahler  
Date: Thu, 9 Sep 2010  
I have been following the discussion about linearizing and thought I should add in my two cents. I teach a variety of levels of physics. I teach 8th graders who are in Algebra 2, 9th graders who barely passed Algebra 1, and seniors who are in AB calculus.

As many of you have pointed out, linearizing can be a very useful tool in a student’s toolbox in helping them develop an understanding of physics concepts.

The first thing I consider, as many of you have, is the math abilities of the students in the class. With my 9th grade class I will not do linearizing. They have enough trouble working with linear graphs that I think trying to do linearizing would confuse them and actually impede their progress of learning physics. There are plenty of modifications to the modeling labs using the Vernier probe ware that you can make where you can have all the data be linear. So for those two reasons I avoid it with those very low math level students.

Secondly, if you are going to do linearizing, do it right. (Sorry on my Soapbox.) As pointed out in Arons Underpinnings, you need to spend the time developing the students’ proportional reasoning skills so they can understand *why* some graphs are linear and other graphs are not. We know from research and our own experiences that our physics students’ proportional reasoning skills are very deficient. These skills I believe are essential for the students to really understand *how* two variables are related to each another and *why* modifying an axis yields a linear graph. Too often I have seen teachers present linearizing using a mechanical approach. Ex. If the graph is an inverse relationship it looks like this. To linearize the graph you do this to the x-axis. This mechanical approach is exactly what we get mad at the math teachers for doing, and I would argue adds little or nothing at all to the students’ understanding of proportions and physics.

Lastly, hold off on using the computer. I have found there is something to making the students do linearizing by hand at first. They develop a better understanding of the process and how it works. Once I am comfortable that they understand linearizing, then we move to using the computers. I try to steer away from letting the students using the non-linear curve fit features.
of LoggerPro/Excel. I have found if you let them use those features, the students go through fit after fit just trying to find the one the “looks” the best. They don’t think about what they are doing.