FRAMING DISCOURSE FOR OPTIMAL LEARNING IN SCIENCE AND MATHEMATICS
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A Dissertation Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy ARIZONA STATE UNIVERSITY May 2007

Keeping an Eye on the Model

A delicate but critical shift of attention: zooming in and zooming out

It is all too easy for students to lose sight of "the model" in the course of routine classroom activity. The initial paradigm lab in which physics students engage in a modeling instruction sequence typically results in an equation, and instead of viewing this equation as *a mathematical representation that encodes the structure of the model* they are building, they sometimes mistake it for the model itself.

If we take the microscope as a metaphor for the student's way of looking at a problem, it would be as if the student zooms to high power in order to see the details of the problem space. Once she has these details in focus, however, she must not forget to zoom back out in order to make sense of the problem by putting what she sees into some sort of 'big picture' context. At times, the student may overlook this critical 'zooming out' step, and this is a place where careful probing by the teacher can help the student refocus, and develop some skill in knowing when and how to zoom in and out.

Lili: The value of the velocity is getting closer and closer to zero.
TEACHER: on the way up.

Lili: On the way up.

TEACHER: What about on the way down?

Lili: it was getting farther and farther from zero.

TEACHER: Well, what do you mean by away from zero?

Lili: The value is more negative.

TEACHER: That's more consistent with what's on the board. [The object is] moving in the negative direction and faster. Dave, were you going to add something to that?

Dave: Oh yeah—I was going to say when it's rolling up the hill it's slowing down but when it starts rolling back it's speeding up.

TEACHER: perfect.
(DHS 9-30-05)

Here the teacher redirects the student's attention from the mathematical to the physical interpretation, and he is assisted by Dave, who translates the teacher's statement to something that everyone should be able to understand.

Knowing when and how to adjust her focus on a problem to a finer or coarser view requires considerable skill on the part of the student.

If only I had a hammer

A whiteboard representation is a composition of symbols and therefore it requires the student to be able to call upon what she knows about the structure of what she perceives. An incomplete symbol set for use in representing a system's structure limits the ways that a student can compose and manipulate a representation.

One ubiquitous finding in my observations of classroom activities was the *"equations are tools"* metaphor, which was evident in the discourse of students and teachers alike. Here are three examples of "equations are tools" taken from two different students and from the teacher discussing whiteboards presented during the same class period:

> *Nancy: So we used the equation of the final velocity equals acceleration times time plus the initial velocity and plugged in negative five meters per second squared for acceleration, seven point oh seconds for time and an initial velocity of twenty meters per second, and we got twenty point five meters per second for the final velocity.*
>
> …
>
> *Peter: For number 4 we used final velocity equals acceleration times time value plus initial velocity. Then we plugged in the given numbers and got 9.05 m/s. Anybody have any questions?*
>
> ….
>
> *TEACHER: So is it appropriate to use an equation from the battery powered car lab? The second problem is…this isn't the change in velocity (he changes the formula he has written on the board—crosses out the delta in front of $v = \Delta x/\Delta t$ and writes the word average above the v) …it's average velocity…watch out for that. That's probably the most common mistake people make on these kinds of problems. They get to one—they're in a rush—they see a distance, they see a time, they want to know acceleration…they use distance divided by time and get a velocity and then use that velocity divided by time and get an acceleration. Does that work here?*
>
> *(DHS 10-3-05)*

The tendency to want to think of equations as both models and tools is not surprising in light of many students' preference to see learning in mathematics and science as a procedural activity. A great deal of both class time and textbook "page count" are given to cultivating procedural fluency. If students view the goal of performing a paradigm lab as finding an equation (rather than identifying a model) then the goals of engaging in the subsequent modeling tasks found on worksheets may become (1) finding the right equation to use, and (2) learning to use it correctly (rather than elaborating and applying the model).

> *Zane: You probably did it wrong…? Here. Let's work it out. Let's work it out. Let's do it on the board.*
>
> *Jimmy: Start by writing the equation.*
>
> *Zane: Yeah.*
>
> *Jimmy: Delta x…*
>
> *(Hannah is talking about food as she writes their names on top of the board. Zane is writing an equation and data on the board: $\Delta x = 1/2at^2$, $\Delta x = 110m$, $t = 1.5s$)*

Jimmy: That's not the whole equation though.

Zane: But we don't need the rest though.

Jimmy: Why?

Zane: There's no…okay whatever…(adds + v_0t to the equation)

Jimmy: Then you just put a zero…

Gui: Just show all the work.

Hannah: This is supposed to be a five… (rubs the 1. off the 1.5 seconds)

Zane: Okay. Whatever. 110m = 1/2a(5s)^2…[ ] squared…

Hannah: make it legible…

Zane: It's legible…I can read it… (pause while he continues to write)…no we'll get a slow… what's the answer?

(Board now shows

$$110m = 1/2a(5s)^2 + (0)(5)$$

$$\frac{110m}{25s^2} = \frac{1/2a(25s^{2)}}{25s^2}$$

$$\frac{\text{١١٠}\,m}{1} \cdot \frac{1}{25s^2}$$

$$\text{٢٠٤}.4\text{m}/s^2 = 1/2\text{a} \cdot 2$$

$$8.8\text{m}/s^2$$

Hannah: 8.8

(DHS 9-30-05)

For Hannah, Jimmy and Gui, the whiteboard exercise recounted above was about choosing the correct equation, using it correctly and performing the necessary computations to find the correct answer.

What teachers want is for students to work together to identify, explore and elaborate fundamental physical relationships, and ultimately generalize them so that they can be re-used. What teachers often get, however, is students working together to identify the necessary equation and use it as a tool to get an answer. They do this because, in their experience, this is what the culture of schooling values—what they can earn points for.

Teachers need to be explicit about valuing models over answers, and in order for these models to be complete, they must have both the conceptual structure and the spatial representation dimensions. Attention to task design, and the framing and scripting that necessarily accompanies this activity, can help them redirect students' attention so that they encode information spatially and well as propositionally.

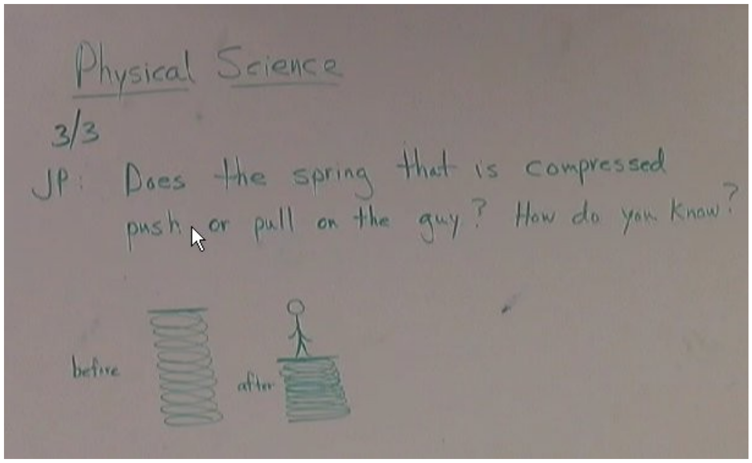

Figure 1. A journal prompt that makes use of both conceptual structure and spatial representation

One way to discover what a student's model contains is to watch and listen for 'what counts' as far as the students are concerned. When their whiteboard shows only an equation and their oral presentation describes only the procedures they used to solve the problem with the equation they chose, the metaphors students frequently use are the *object collection* (*container*) and object construction metaphors described by Lakoff (2000). Verbalizations that make use of these metaphors are often accompanied by the expression "plug and chug". When employing a *container* metaphor students describe the collection of pieces of information from the problem space. When employing the object *construction* metaphor, they talk about plugging things into an equation to get and answer (the thing they think they are supposed to construct).

Lili: We forgot to <u>put</u> a direction, but it says that…it says to assume the average mass of the riders is 75 kilograms so if you've got 20 people you have 1500 kilograms in the elevator...they say to <u>make</u> the mass of the elevator 500 kilograms so the total mass is 2000 kg…they said the max force that the cable can support is 30,000 Newtons, and so you just <u>use</u> that the[equation] net force is equal to "a" times "m". We said that the max force is the net force... [inaudible] acceleration…so you <u>put</u> in 30,000 Newtons equals acceleration times 2000 kilograms which <u>gets</u> you 15 meters per second squared.

(DHS 11-15-05)

Here Lili makes a valiant (but misguided) attempt to *construct* (note the verbs used: make, put, use, get) an answer to an elevator problem that asks her to determine how fast an elevator will accelerate using the equation $F_{net}=ma$ and substituting mass and force values given in the problem to find an acceleration.

Brought along v. Brought about

In cases such as the ones described above, it can be telling to hear what counts as "understanding" in the view of the students. Often "understanding = *getting* the right answer," i.e. demonstrating the correct *object construction* procedure. Again, the tendency to define understanding as procedural fluency is something that students bring along with them as a part of the culture of schooling that they have experienced up to this point.

Zane: So what did you guys get for the acceleration on 7c

Jimmy: 7c?

Hannah: I got negative one.

Jimmy: negative 8.57

Hannah: Oh whoops (flips page). Yea I have that too! Ya see I am understanding this, I just get stressed out and people make me feel like I don't know what I'm doing.

Jimmy: (Clapping) Yea Hannah! You got the right answer on the last one.

Zane: I'm just so happy I finally understand everything.
(DHS 10-5-05)

The teacher must be alert for what is missing (this is not easy!) from students' model description in order to bring about an expanded definition of understanding that includes both conceptual structure and spatial representation.