Graphs as a Problem-Solving Tool in 1-D Kinematics

Dwain M. Desbien, Estrella Mountain Community College, Avondale, AZ

In this age of the microcomputer-based lab (MBL), students are quite accustomed to looking at graphs of position, velocity, and acceleration versus time.[1] A number of textbooks argue convincingly that the slope of the velocity graph gives the acceleration, the area under the velocity graph yields the displacement, and the area under the acceleration gives the change in velocity.[2] I also use dimensional analysis to help make the arguments. While students are very often encouraged to sketch graphs of motion to help build a better understanding,[3] too little emphasis is placed on the fact that such graphs can be used to actually solve problems. This paper shows two examples of how to use sketches of velocity and acceleration graphs to solve 1-D motion problems.

A simple problem that can easily be solved using graphs is:

1. A ball is thrown vertically downward from a 120-m high building. The ball hits the ground in 2 seconds. How fast was the ball thrown?

Sketches of the velocity and acceleration graphs are shown in Fig. 1. For simplicity, the downward direction is assumed to be positive, and, for ease of calculations, the freefall acceleration g is taken to be $10\ \text{m/s}^2$. Knowing that the area under the acceleration graph is equal to the change in velocity (Δv) of the object yields

$$\Delta v = gt = 10\ \text{m/s}^2 \times 2\ \text{s} = 20\ \text{m/s}. \quad (1)$$

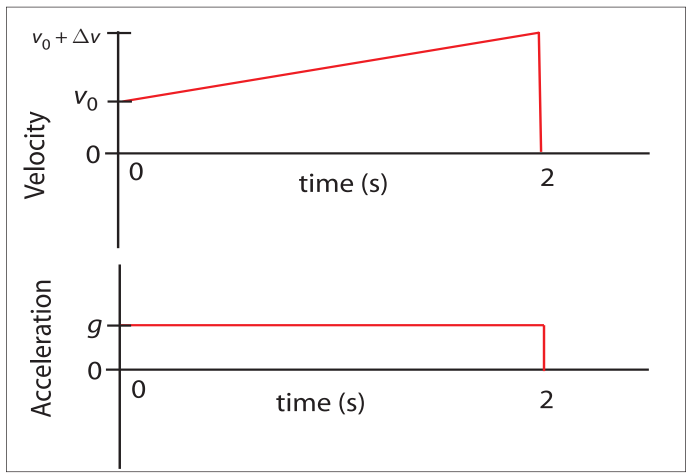

Fig. 1. Velocity and acceleration graphs for Problem 1.

The area under the velocity graph is the displacement, which was given to be 120 m. Breaking the area under graph into a rectangle and a triangle yields

$$120\ \text{m} = v_0 \times t + \tfrac{1}{2}\,\Delta v \times t$$
$$= 2\ \text{s} \times v_0 + \tfrac{1}{2}\,20\ \text{m/s} \times 2\ \text{s}. \quad (2)$$

One can easily solve this for the initial velocity of the object, thus obtaining the answer to the problem (50 m/s is a quite unrealistic initial velocity for a thrown ball). No explicit use of the standard kinematic equations is made; the solution is based on only the graphs.

The above problem is relatively simple; to see the real power of the method requires a more difficult

1-D kinematics problem. While working on my PhD at Arizona State University, I was helping with a class in which the following homework problem[4] was assigned:

2. Determined to test gravity, a student walks off the CN Tower in Toronto, which is 553 m high, and falls freely. His initial velocity is zero. The Rocketeer arrives at the roof of the building 5 seconds later to save the student. The Rocketeer leaves the roof with an initial velocity downward and then is in freefall. In order both to catch the student and to prevent injury to him, the Rocketeer should catch the student at a sufficiently great height and arrive at the ground with zero velocity. The upward acceleration that accomplishes this is provided by the Rocketeer's jet pack, which he turns on just as he catches the student; before then the student is in freefall. To prevent discomfort to the student, the magnitude of the acceleration is limited to five times gravity. How high above the ground must the Rocketeer catch the student?

As I was grading the assignment, one student's solution stuck out. When I first saw it, I was convinced that something must be wrong as the problem had been difficult for me and could not be that simple. I was wrong.

The simple solution consists of sketching the velocity and acceleration graphs for the falling student (Fig. 2; down is taken as the positive direction) and using a bit of reasoning. Since the overall change in the student's velocity during his motion is zero, the total area under the acceleration-versus-time graph must equal zero. This area consists of a positive part (above the time axis) and a negative part (below the time axis). The two rectangular areas must have equal magnitude, and since their heights differ by a factor of 5, so must their widths (the two corresponding times). Therefore, the freefall time is five times longer than the time for slow-down to rest. Now, looking at the velocity graph, the area labeled 1 corresponds to the displacement while in freefall, and the area labeled 2 represents the displacement after the Rocketeer has caught the student. The triangles forming areas 1 and 2 have the same height, and the base (time) for area 1 is five times the base (time) for area 2. Therefore, area 1 must be five times larger than area 2. This means that area 2 must be 1/6 the total height of the building, or 92.2 m.

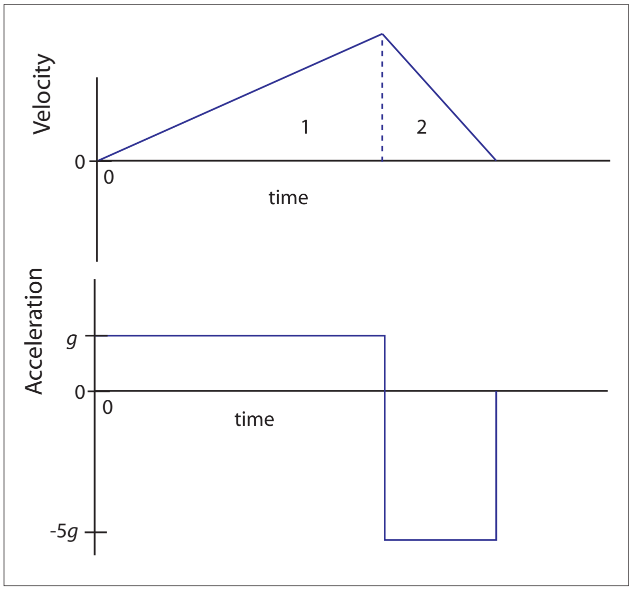

Fig. 2. Velocity and acceleration graphs for Problem 2.

After studying this solution for a long time and finding no flaws in the physics, I pulled out my physics book and tried solving other one-dimensional kinematics problems in a similar manner. It quickly became obvious that they could be done using graphs, and in most cases this is the easier method. I was hooked and began teaching one-dimensional problem solving this way in all my classes.

The next class period I asked the student whose solution this was why he chose to do the problem this way. He explained that we had emphasized graphs so much in class that they must be more useful than something we are simply supposed to sketch for the problems. I was amazed and appreciative as I now had a new way to teach problem solving.

This kind of solution is more visual and helps get students away from hunting for the "correct equation." It has worked well for me at all levels of introductory physics, conceptual through calculus based. If you find other interesting problems that are especially suited to the above method, I would appreciate your sharing them.

References

1. For example R.K. Thorntonand D.R. Sokoloff, "Learning motion concepts using real-time microcomputer-

based laboratory tools," *Am. J. Phys*. **58**, 858–867 (Sept. 1990).

2. For example, J. Wilson, A. Buffa, and B. Lou, *College Physics,* 6th ed. (Prentice Hall, New York, 2007), p. 49.
3. For example, R. Knight, *Physics for Scientists and Engineers: A Strategic Approach,* 2nd ed. (Addison-Wesley, Reading, MA, 2007), Chap. 2, and *Minds on Physics* (http://umperg.physics.umass.edu/resources/mop).
4. A problem as it appears in H. Young and R. Freedman, *University Physics,* 9th ed. (Addison-Wesley, Reading, MA, 1996), p. 59.

PACS codes: Editor

Dwain Desbien *is division chair of the Math and Science Division at Estrella Mountain Community College. He is active with the Two-year College Physics Workshop Project. Currently, he is also a member of the AAPT Executive Board.*

Estrella Mountain Community College, 3000 North Dysart Road, Avondale, AZ 85392; dwain.desbien@emcmail.maricopa.edu