

Making Work Work

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February 22, 2005

Abstract

Some of the difficulties that students have in dealing with work and energy can be addressed through energy flow diagrams and energy bar graphs which allow students to represent features of systems and processes that $W = F\Delta x \cos \theta$ cannot. These representations are applied to various physical situations. This paper was prepared for the Physics Modeling Workshops in the summer of 1998 and revised on February 22, 2005.

1 Introduction

In many introductory physics texts the concept of work is separated from the concept of energy and from the concept of system in such a way that certain difficulties arise. Work is a carefully defined and subtle concept that is dependent on and derived from the energy concept and the system concept. The purposes of this paper are to identify certain difficulties that arise from common treatments of work and then to present some representational tools that can help to prevent these difficulties.

2 Difficulties with common approaches to energy

Often work is defined as

$$W = F\Delta x \cos \theta \tag{1}$$

where $F \cos \theta$ is the component of the constant force F acting on some object in the direction of the one dimensional displacement of the point of application of the force, Δx . Three problems for students can arise when this definition of work is not closely linked to the concept of energy. They are: 1) the belief that forces do work, 2) the belief that every force acting to some extent along the direction of an object's motion must do work on the object, and 3) the belief that the work done by an agent exerting frictional forces can be calculated by equation (1). Sadly, the understanding that work is an amount of energy that flows into or out of a system can be lost on students who know work only as $W = F\Delta x \cos \theta$.

The claim that forces themselves do work arises from the separation of the concept of work from the first law of thermodynamics,

$$\Delta E = Q - W \quad (2)$$

in which ΔE is the change in energy of a system, Q is the energy gained by the system through heating, and W is the energy lost from the system through working. A good understanding of $\Delta E = Q - W$ informs us that work has meaning only with respect to explicitly defined systems. This is not clear from $W = F\Delta x \cos \theta$ alone. Only systems that can contain and exchange energy can perform work. Forces are not systems and cannot contain or exchange energy, and, therefore, should not be said to perform work, although it is commonly said that forces do work.

Another problem with thinking of work only as $W = F\Delta x \cos \theta$ is that it does not explicitly represent an exchange of energy *from one system to another*. Therefore, questions like “How much work is done when a 50 kg person climbs a 100 m hill?” are not specific enough to answer. Are we talking about work done by the gravitational field or by the person? The work done by the person is always greater than the energy gained by the gravitational field, so it makes a difference. We need to enable our students to build a better understanding of physical situations than that fostered by such questions. Because the quality of our understanding depends on the quality of our representational tools, it will be of benefit to students to have ways to represent systems and the flow of energy between and within systems. Two useful ways to represent energy exchanges have already been developed and will be described below. They are energy bar graphs and energy flow diagrams. After the students are acquainted with energy exchanges through the use of such representations, the concepts of working and heating can then be understood as particular kinds of already familiar exchanges.

Another difficulty common among students is the belief that work is done through *any* force on an object acting at least somewhat in the direction of its motion. For an example of a situation in which this belief is misleading consider a self-propelled object such as a jumping disk. The jumping disk is cocked into position and placed upon a table. No energy enters the jumping disk from the outside world as it pops up into the air. It does move upward, because the table exerts a force on it that causes it to accelerate. But there is no work done on the disk. There is simply a redistribution of energy within the disk itself. A careless application of $W = F\Delta x \cos \theta$ or the work-energy theorem ($W = \Delta KE$) will lead students to the conclusion that there is work done on the disk. There is no work done on the disk because of the “jump.” This case is discussed further in Section 3.4.

Yet another problem with the work concept actually is sometimes even prescribed by textbooks. Work done by agents exerting frictional forces cannot be calculated by $W = F\Delta x \cos \theta$. Despite this fact, many textbooks actually assert that it can be calculated this way. The problem here is our limited knowledge. We do not know the actual magnitudes of the forces being exerted on the microscopic level at the points of contact, nor do we know the distances over which those forces are exerted. Textbooks often use the net frictional force and the center of mass displacement Δx_{cm} as though they are the appropriate variables to insert into $W = F\Delta x \cos \theta$. They are not. Therefore students end up with misleading results as they use $W = F\Delta x \cos \theta$ to solve friction problems, and for no good reason, since often the desired results could have been properly obtained through the use of Newton’s Second Law.

3 Describing and representing energy in systems

A careful treatment of energy will keep track of the amounts of energy stored in various entities and will also explicitly identify the energy source and the energy recipient involved in every pertinent energy exchange. Keeping track of the amounts of energy in various entities can be done quite nicely using the energy bar graphs advocated by Alan van Heuvelen.¹ Keeping track of the sources and recipients of energy can be done with energy flow diagrams as prescribed by the faculty of the Department of Physics Education at the University of Karlsruhe, Germany.² Although the Karlsruhe group has developed a thoroughly integrated physics curriculum using energy as the fundamental and unifying concept, we will pursue only a small portion of their agenda.

In many introductory courses, energy is characterized as thermal energy, kinetic energy, gravitational energy, electrical energy, elastic energy, *etc.* In the face of these commonly used names, it is important for students to understand that energy does not come in different forms. The distinctive names arise because of the different systems in which energy is stored, not because there are different forms of energy. Energy is just energy. We will attribute, for example, thermal energy to the “thermal” system in an object, that is, to all its atomic scale oscillator systems, translational kinetic energy to the center-of-mass system of an object, elastic energy to the structural system, gravitational energy to the gravitational field, and electrical energy to the electric field. Different “storehouses” result in the different names, not different “kinds” of energy.

It will also be noticed that the term potential energy is not used in this paper in connection with any way of storing energy. This is done for two reasons. First, it is sometimes said that there is kinetic energy and then there is energy that is stored, namely potential energy. Actually all energy is stored in some physical entity, not just the so-called potential energies. For example, kinetic energy is stored in moving particles and in flywheels, *etc.*, and is available for transfer. Therefore, the characterization of some kinds of energy as potential (meaning stored) and kinetic (not stored?) is artificial and possibly misleading. Second, potential energy is sometimes contrasted with actual energy as though potential energy is not real energy like, say, thermal energy is real energy. Therefore, sometimes potential energy is thought of as only potentially energy. Of course, potential energy is just as real as energy stored in any other way.

It is interesting to point out that gravitational energy cannot be considered to be stored in an elevated object. Students often come to believe that an object itself can possess gravitational energy in the same sense that it can possess kinetic energy or thermal energy. However, whenever the amount of energy in some entity is changed or redistributed, one or more state variables of that entity will change. Since no property at all of an object itself changes as it is raised or lowered, gravitational energy cannot be regarded as being contained in the object. Some argue that gravitational energy is a property of the system consisting of the two gravitating bodies under consideration and that their separation constitutes the state variable that changes with the amount of gravitational energy. All well and good. But this claim simply skirts the issue of where in the system the energy is actually stored. If the system consists of only the two gravitating bodies, and neither contains the gravitational energy, there would seem to be no place for the energy to exist.

¹Alan van Heuvelen, *ActivPhysics*, Addison-Wesley (Reading, MA, 1995).

²G. Falk and F. Herrmann, eds., *KonzepteeineszeitgemässenPhysikunterrichts*, 5 vols., (Schroedel Verlag, Hannover, 1982).

The missing part of the system, the gravitational field, needs to be recognized. We will consider gravitational energy to be stored in the gravitational field. More on the gravitational field as a container for energy can be found in an article by Stössel.³

If we claim that energy is stored in the gravitational field, and if a change in energy content is indicated by some change in the thing that stores it, then what changes in the field when it gains or loses energy? In the Newtonian picture of gravitation, the strength of the gravitational field changes. For example, if a meteor falls to earth there is less gravitational energy than before it fell. But now the earth has a bit more mass and a slightly stronger gravitational field. The stronger the gravitational field, the less gravitational energy it possesses! The enormous gravitational field of a black hole is the symptom of an enormous loss of gravitational energy. In general relativity the curvature of space-time depends on the position of objects in space. This curvature, then, is what changes when energy flows to or from a gravitational field. Describing energy storage in gravitational fields, however, remains an outstanding problem in general relativity.^{4 5} This is a nice example of how close the concepts in introductory physics can come to the cutting edge of knowledge.

Why bother with something like the curvature of space-time that students cannot possibly measure? We bother because something is gained, and that is the ability to visualize energy flowing from one thing to another, producing changes in the giver and in the receiver. This enables students to describe energy transfers in terms of meaningful physical entities and the variables that describe them rather than in terms of the misleading and finally meaningless notion of energy changing from one form to another.

3.1 Energy bar graphs

Energy bar graphs enable one to describe the state of some system in terms of the energy stored in various physical entities within the system. To make an energy bar graph one first identifies the system of interest. The system is usually composed of entities that are involved in energy transfers. The system is examined in an initial state and in a final state. In the energy bar graph for the initial state, the energy stored by each entity is represented by a vertical bar along with another bar to represent any energy that flows into or out of the system through working or heating as the system progresses from the initial state to the final state. In the final state the energy stored by each entity in the system is represented in a new bar graph. In any case, the total amount of energy in the final state of a system must equal the total amount of energy in the initial state plus or minus any energy that was gained or lost through interactions with entities outside the system.

³W. Stössel, “Die Rolle der Energie in der Mechanik,” in Falk and Herrmann, eds., *Konzepteeineszeitgemaessen-Physikunterrichts*, (Schroedel Verlag, Hannover, 1982), Vol. V, pp. 48 ff.

⁴Misner, Thorne, and Wheeler, *Gravitation*, (Freeman, San Fransisco, 1973), pp. 466 ff., pp. 603 f.

⁵S. Hayward, Quasi-local gravitational energy, *Phys.Rev.D*,**49** (1994) 831-839.

3.2 Energy flow diagrams

The energy flow within and between systems can be represented by energy flow diagrams. Such diagrams will illustrate how a system gets from its initial state to its final state. To make a flow diagram, things that are involved in the interactions of interest are identified and represented by named rectangles. Each rectangle, then, represents an energy storehouse, something that gains or loses a significant amount of energy in the course of the interactions. These rectangles correspond to the bars on the energy bar graphs. The flow of energy from one storehouse to another will be represented by an arrow or arrows, the width of which can be used to serve as an indicator of the amount of energy transferred during the transition from the initial state to the final state.

The difficulties associated with the sign convention for work W done by or performed on a system are avoided for beginners by concentrating only on determining whether energy was gained or lost by the system of interest.

3.2.1 Energy bar graphs and flow diagrams involving the free particle model

We will first consider the case of a person pulling a box with a horizontal rope along a rough surface at a constant speed. Such an object is in the domain of the free particle model; it is subject to zero net force. A careless but frequently seen application of $W = \Delta KE$ will lead a student to conclude that no work is being done on the box, because the change in the kinetic energy of the box is zero. Or a student may calculate the work done on the box by the person according to $W = F\Delta x \cos\theta$ and then calculate the work done through the force of friction on the box in the same way. Since the frictional force is equal and opposite to the force exerted by the person, the student, using the same Δx for the frictional force as for the force exerted by the person, will find that the frictional work just cancels that done by the person.

This, of course, leads to inconsistencies that ought to bother a student. The box becomes warmer, indicating that it has gained energy. Therefore the work done on the box must be greater than zero. If the student realizes this, then instead of trying to figure out how one can properly use $W = F\Delta x \cos\theta$ for the frictional force (not a trivial task!), or using the work-energy theorem even though it is not valid for this system, he could use energy bar graphs and flow diagrams to characterize the system qualitatively, more accurately, and more cogently.

Let us use energy bar graphs and an energy flow diagram to represent the person dragging the box in Figure 1. First, we should specify the system of interest. This is a difficult task for most students. It may not be clear what one should include or exclude. One way to approach this task is simply to try to identify what things exchange energy. This may be determined by identifying things that change in some way. We will define the system to consist of the person, the box, and the floor, all of which are changed by this process. There are two different subsystems of the box that store energy in this example. One is the center-of-mass subsystem which stores kinetic energy. The other is the thermal system which stores thermal energy. Therefore, we sketch four boxes, one for the person, two for the box (one for each subsystem), and one for the floor. Second, we should describe the initial state of the system, *i.e.* when the box has just begun to slide with constant speed. The initial state of the system is represented by the energy bar graph on the left. We include

bars for each of the things in our system. The person has some amount of energy, and the box has some kinetic energy. Third, we must describe the final state of the system. The box has been sliding at constant speed for a while. The final state is represented by the graph on the right. The person is a bit depleted from all that hard work of dragging this box, the box still has the same kinetic energy we are told, and now the box and the floor have been warmed some by the friction between them. Note that the box has two bars in the final state graph. That is because the box has two subsystems of interest, its center-of-mass subsystem to which we attribute kinetic energy and its thermal system by which we mean all the atomic and molecular oscillators in the box. Fourth, we can represent the process by which the system goes from the initial state to its final state by sketching arrows for the energy transfers.

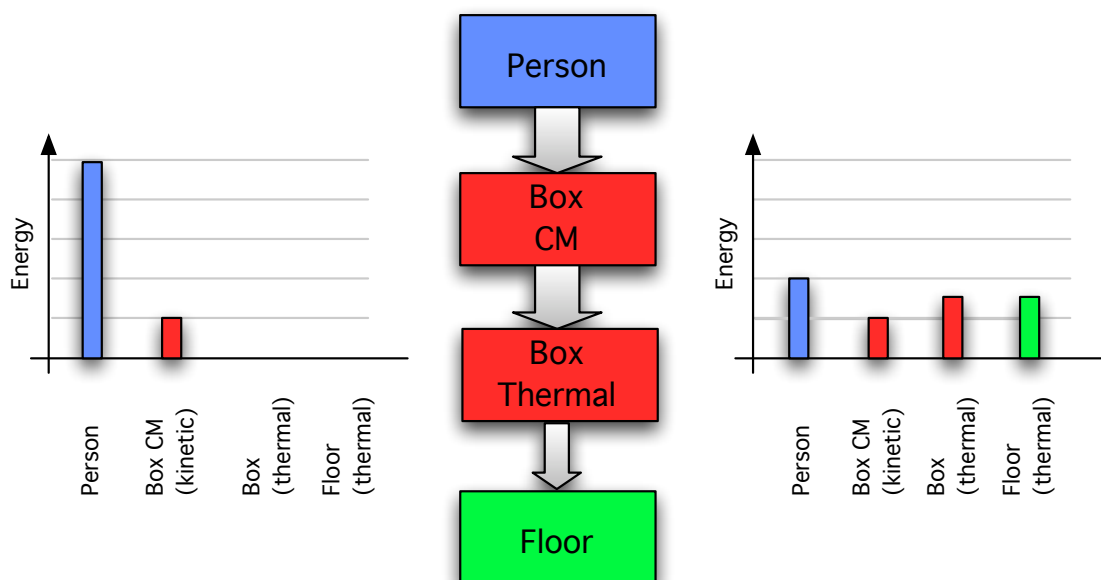


Figure 1: Energy bar charts a flow diagram for a box being pushed across the floor

This energy flow diagram is by no means the simplest. The energy put into the box by the person is transferred at innumerable locations along the rubbing surfaces to the molecular structures of the box and of the floor which we then call “thermal” energy. Some of the energy entering the box from the person stays in the thermal system of the box while some goes to the floor, and both become warmer. This process is depicted by representing the energy flow to the thermal system of the box as a thicker arrow and the energy flow from there to the floor as a thinner arrow. The box contains more energy at the end of the process than at the beginning, even though it maintains constant speed. Students wonder at what point they should consider the warming of the environment as the box and floor cool off. After all, if one waits long enough after stopping, the whole thing will cool off. The final situation is a matter to be determined by the goal one has for a particular energy description. Often it needs to be made clear to the students to avoid too much ambiguity.

It is worth seeing how the energy bar charts change if one considers the box alone to be the system of interest (Fig. 2). One could indicate this choice on the energy flow diagram by enclosing the diagram for the box with a dashed line to represent the system boundary. The graphs and diagrams

for the person dragging the box might be sketched something like Figure 2.

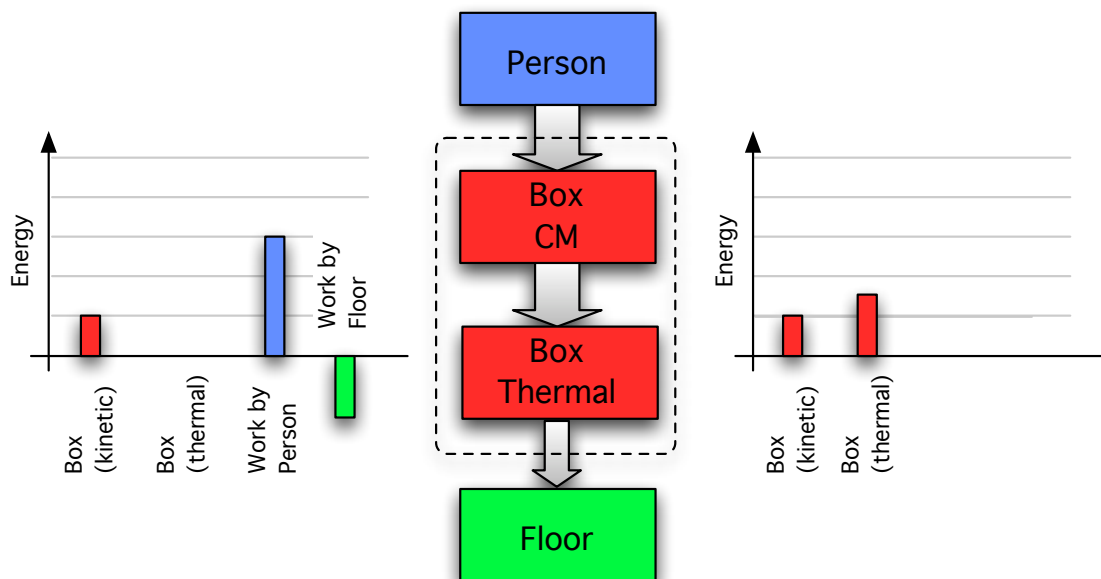


Figure 2: Identifying the box itself as the system of interest

As mentioned above, there is a tendency for students who deal with work exclusively according to $W = F\Delta x \cos \theta$ to claim that there is no net work done on the box. Many textbooks will make the same claim. However, net work *is* done on the box. Using the energy flow diagram, we can see that energy flows across the system boundary to the box because of a force from the person. Energy entering a system in this way is called work. It is not necessary to call it work in order to understand what is occurring, however. Also, energy flows across the boundary from the box to the floor because of frictional force, and the floor becomes warmer as a result. This quantity of energy is also “work.” And because some thermal energy remains in the box as indicated by an increase in temperature, the box has more energy than before. The amount of energy gained by the box is the net amount of work done on it. The error of claiming that no net work is done on the box is avoided, although there is no quick and simple way to find out how much work actually is done on the box by the floor through frictional force.

Why does $W = F\Delta x \cos \theta$ fail to tell us the work done by the floor on the box? There must be something subtle about how one uses this relation. These subtleties are discussed in some detail by Sherwood and Bernard.⁶ It turns out that the details of how the frictional force arises on the microscopic scale complicate the calculation of work done through friction. At the actual points at which forces are exerted we know neither the sizes of the actual forces nor the distances over which they are acting. Furthermore, $W = \Delta KE$ and $W = F\Delta x \cos \theta$ are defined for a particle, and a particle can have no internal energy, because particles, by definition, have no structure. $W = F\Delta x \cos \theta$ should not, in general, be used for work done through frictional forces, because a particle model is not a valid model for objects that have changing internal energy and, therefore,

⁶B. A. Sherwood and W. H. Bernard, “Work and heat transfer in the presence of sliding friction,” *Am.J.Phys.*, **52**, pp. 1001-1007 (1984).

internal structure.

Other energy flow diagrams can be simpler to draw and interpret. The next example (Fig. 3) has to do with a box that is being lifted vertically at a constant speed by a motor. Energy flows from an electrical powerplant mainly to the environment through heating and also across the system boundary because of electrical forces inside the motor (so it is called work) and then to the box being lifted, but the energy is not stored in the box, which continues to rise with a constant speed. The box is not changed in any way as one would expect were it to gain or lose energy in some way. Where, then, is the energy stored? It is stored in the gravitational field itself. We will call this energy gravitational energy. We avoid calling this energy the potential energy *of the box*, because in no way can this energy be considered a property of the box in the same sense that its kinetic energy can be.

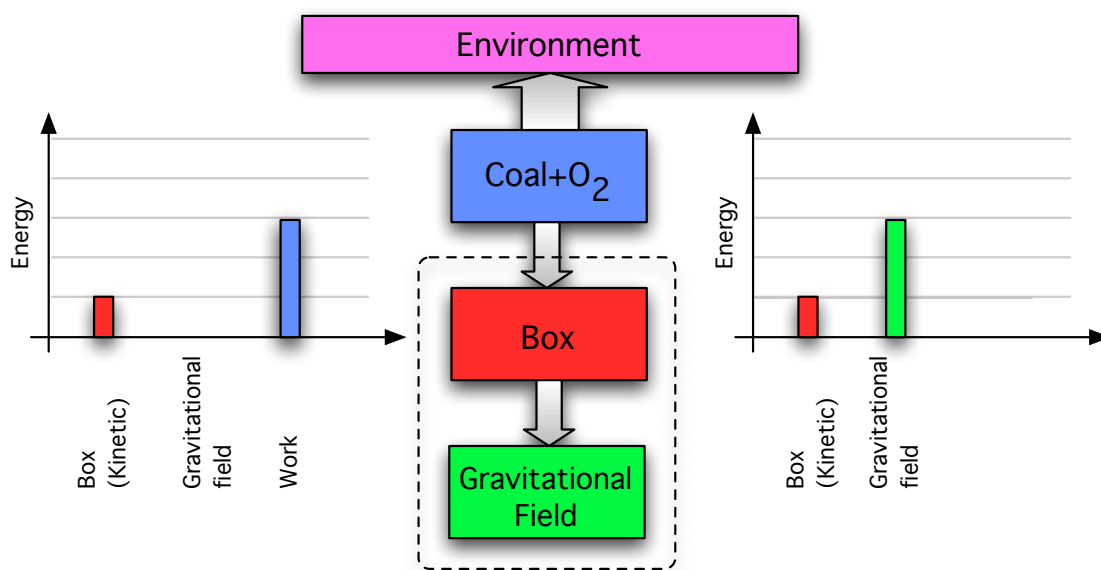


Figure 3: Energy bar charts and energy flow diagram for a box lifted by an electric motor

3.3 Energy flow diagrams for the constant force particle model

Let us consider an object subject to a single, constant force (but not friction!). In this case energy flowing to or from the object will be stored in the object as kinetic energy. For a freely falling object this energy will be exchanged with the earth's gravitational field (Fig. 4). In this case energy is transferred from the gravitational field to the object where it is stored as energy of motion, kinetic energy.

In the case of a bicycle stopping (Fig. 5) we have the kinetic energy stored in the bike being transferred to the caliper brakes and rims of the wheels where it is stored in the atomic and molecular structures (thermal energy). One could design the desired representations of the process as follows: First, specify the system; second, describe the initial state of the system; third, describe

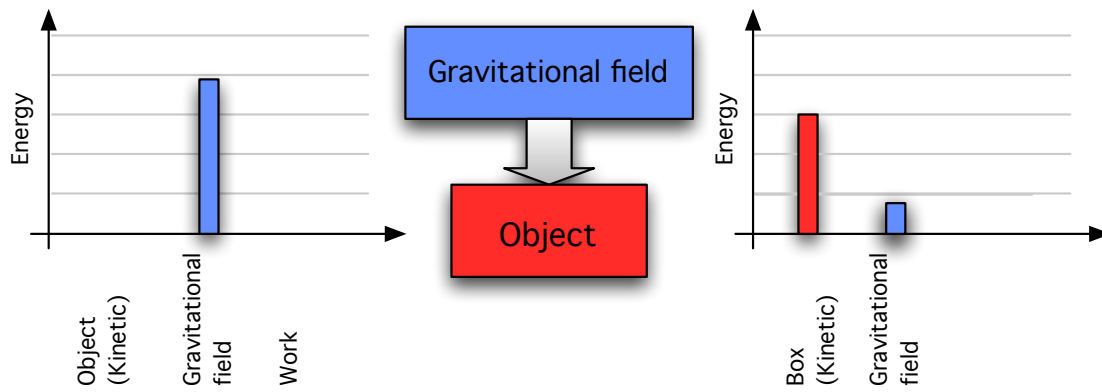


Figure 4: Energy bar charts and energy flow diagram for a freely falling object

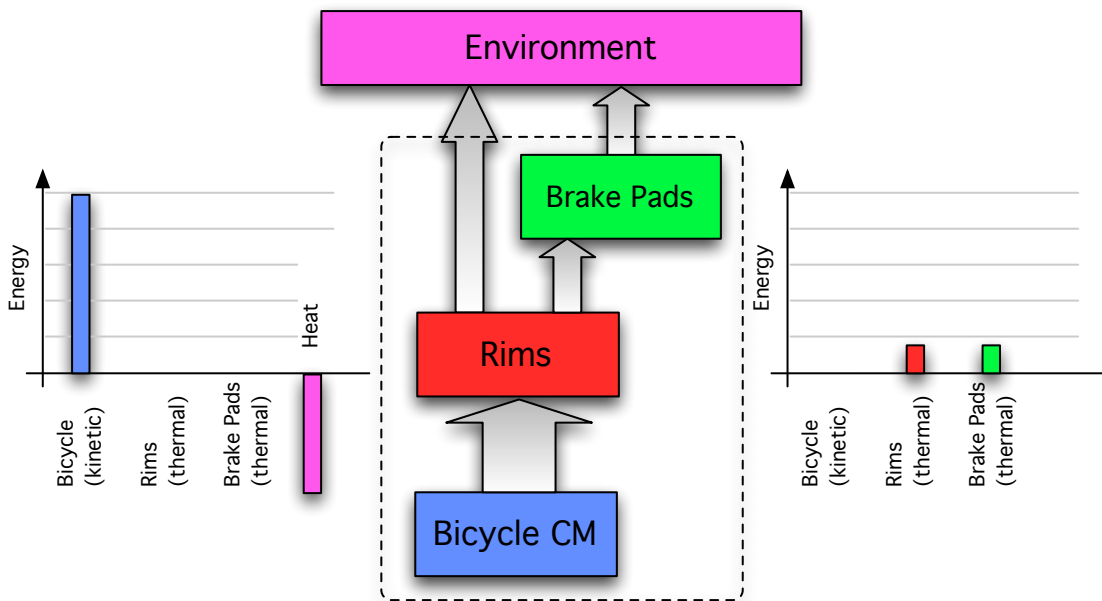


Figure 5: Energy bar charts and energy flow diagram for a bicycle braking to a stop

the final state of the system; and fourth, sketch how energy flows during the process.

First, we identify the systems of interest. The system of interest may be just the bike, but the bike has subsystems that are clearly of interest, too. As mentioned, the brake pads and the wheel rims are going to warm up, so we will include them as subsystems of the bike. The kinetic energy is attributed to another subsystem of the bike, the center-of-mass subsystem.

Second, we describe the initial state of the system in the left energy bar graph. In the initial state the bike is moving along before braking. Third, the final state of the system is described in the energy bar graph on the right. In the final state, the bike has just stopped. The process through which it changes to the final state is represented in the flow diagram. As the bike stops, energy flows from the brakes as they cool to the air where it disperses to more and more molecules. Eventually the rims and brake pads will cool off, all the energy having entered the environment. Exactly how you define the final time will determine how the final state graph looks. An instructor may want to make explicit the final situation for which to draw the final state graph.

The quantity of energy that leaves the brakes and rims as they cool and flows to the air (crossing the system boundary) is called heat, but it is not necessary to call it heat in order to understand the situation. It is interesting to note that in the absence of skidding there is very little work done by the bike, and we neglect it here. If the work-kinetic energy theorem were used here, a student might think that a lot of work is done via the force of the road on the bicycle. But the work-kinetic energy theorem is not valid here, because the bicycle has structure; it is not a particle. In fact, a student should note that not much energy flows to the road at all!

3.4 Energy flow diagrams for the jump of a jumping disk

Earlier in this paper it was pointed out that “jumping disks” have no work done on them, even though they have a significant change in kinetic energy. In order to represent a disk that jumps we will follow the prescribed program: First, identify the things that are involved in energy exchanges. In the case of a jumping disk we know that there is a change in the kinetic of the disk. We attribute translational kinetic energy to the center-of-mass system of the disk. From where does the CM system acquire this energy? It is stored in the strained structure of the disk, so we will attribute the energy stored before the jump to what I will call the “structure” system of the disk. That is all we have to consider in accounting for the jump. Second, identify the initial state: It is when the disk is “cocked” and resting on the table. Third, the final state is just after the disk snaps back into shape and before it has appreciably risen. Fourth, sketch the arrows on the energy flow diagram. This prompts us to sketch graphs and diagrams something like Figure 6. The energy flow diagram makes it easy to see why there is no work associated with this system, because no energy enters or leaves it. Energy is merely redistributed within it.

3.5 Energy in electric circuits

That the energy concept is of fundamental importance throughout all of physics goes without saying. The same representations that we have described can be used also in other branches of

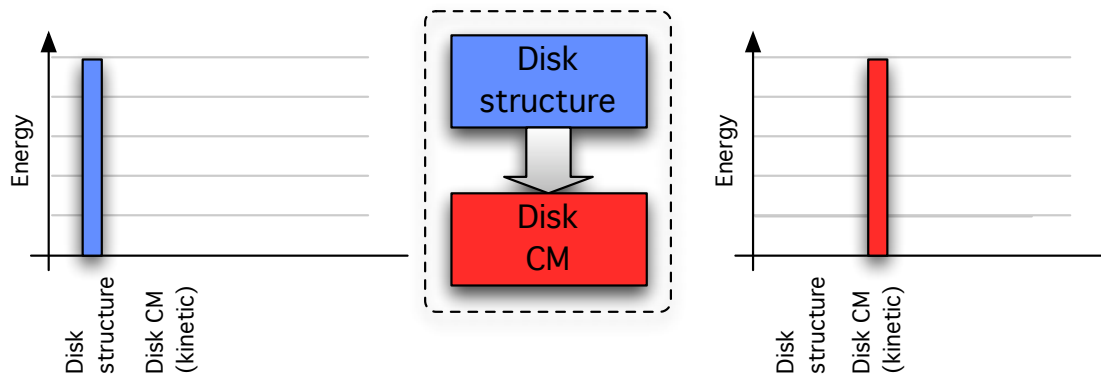


Figure 6: Energy bar charts and energy flow diagram for the “jump” of a jumping disk

physics. Let us look at how flow diagrams can be used in a simple electric circuit consisting of a dry cell, copper wire, a lamp, and a resistor connected in series.

Usually such circuits are not thought of as being in well defined initial and final states as mechanical systems in introductory physics often are. In general, the flow of current is an ongoing process and so is particularly well suited for representation by an energy flow diagram.

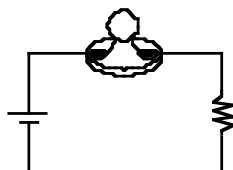


Figure 7: A simple circuit

Notice that the energy flowing around the circuit diminishes as it encounters resistive elements. Students commonly and mistakenly assert that the charge current itself is consumed as it flows around the circuit. It is actually the flowing energy that is “consumed,” by which we mean it is transferred from the circuit system to the environment. This is a case of decent intuition attaching itself to the wrong concept.

It is instructive to examine how energy enters the lamp. It comes from the dry cell, and while it may be useful to say that the electric current carries it, it turns out that the electromagnetic field created by the dry cell, the charges, and the current in the circuit is the most consistent way to account for the energy leaving the dry cell and arriving at the circuit elements. This energy is work that is done on the circuit elements, because the electromagnetic field exerts forces on the charge carriers in the circuit. On the other hand, some of the energy leaves the circuit because it is warmer than its surroundings. We call this heating. Some of it leaves via the light emitted, too. This is a method of energy transfer not mentioned in the First Law of Thermodynamics (Equation (2)). To make this law general enough to fit our circuit we have to introduce radiation as a way

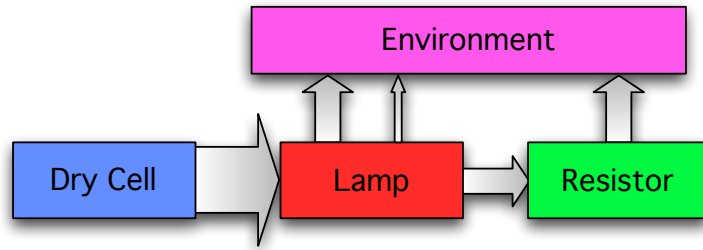


Figure 8: Energy flow diagram for a simple resistive circuit

for energy to enter or leave a system along with W and Q . Let R represent the amount of energy that is gained by a system through radiation. Therefore, we sketch two arrows from the lamp to the environment to account for both the heating and the much smaller flow of radiation.

$$\Delta E = Q - W + R \quad (3)$$

4 Summary

Student difficulties with the work concept can develop as a result of dealing with work only as $W = F\Delta x \cos \theta$. Since understanding work requires a working understanding of energy and systems, students should first become acquainted with how energy flows in the systems that are of interest in introductory physics. The first law of thermodynamics, $\Delta E = Q - W$, provides a deeper and simpler insight into the nature of both energy and work than $W = F\Delta x \cos \theta$. Energy bar graphs and flow diagrams provide students with richer means for representing energy processes, since they illustrate both energy conservation and work. $W = F\Delta x \cos \theta$ can be applied when it can be used in a straightforward manner and where subtleties as with frictional forces do not preclude its use. In this way students can begin to understand not only the appropriate use of this relation but also its practical limitations. The top billing, however, must go explicitly to energy storage and exchanges.