Effect of Modeling Instruction on Development of Proportional Reasoning II: theoretical background

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Abstract

Proportional reasoning is the ability to compare ratios or the ability to make statements of equality between ratios. Piaget considers the ability to reason proportionally to be a primary indicator of formal operational thought, and this stage is viewed as the highest level of cognitive development. Proportional reasoning is taught primarily in mathematics courses, but student success in secondary science is highly dependent on proportional reasoning ability.

The focus of this study was to compare traditional math instruction to instruction that is consistent with the Modeling Method of teaching physics and instructional suggestions from proportional reasoning research. Part I is the empirical study. Part II is theoretical background and justification for methods used.

I. PIAGET'S WORK

Although Piaget's work was conducted over thirty years ago, from it, fruitful research has been conducted, awakening serious implications for math and science education reform through curriculum design. In order to understand its implications, a brief review of Piaget's work and related literature is necessary.

Piaget describes the developing intellect through a series of stages. Children seem to progress very systematically through Piaget's sensory motor and operational stages to what is known as *concrete thought.* The transition to *formal thought* was believed by Piaget to be achieved during early adolescence (Raven, 1973). Formal thought is considered to be the highest stage of intellectual development. Lawson (1975, p. 348) describes it as a stage where "meaning is given to these (formal) concepts not through the senses, but through imagination, or through their logical relationships within the system." Driver (1978) further clarifies this stage by indicating that it is completely free of context. Contrary to Piaget's belief, Kolodity (1977) states that a majority of college freshmen do not function at this level and Lawson (1975) cites that 40%-75% of post-secondary students do not operate at Piaget's formal level.

Driver (1978) suggests also that the transition into the formal stage is not clear-cut. Identification is task dependent and there is a range of variability as to the level of operation for an age group. This is evident in quantitative research where definitions of transitional stages between concrete and formal operational levels were identified (Lawson, 1975). Bliss (1995) seems to think that Piaget may have turned formal operational thought into a "false God"(p.151). Bliss (1995) and Driver (1978) continue to cite problems with Piaget's theory in that the clinical method that Piaget used to generate his theory is not quantitative and the formal aspects of the theory are not clearly defined. Many of these arguments stem from the fact that Piaget's theory is philosophical in nature rather than scientific.

This, however, does not mean that Piaget's theory should be ignored. Rather, serious considerations should be made before putting it into educational practice, especially when considering the transition from concrete to formal thought. Arguments against Piaget's theory concerning these levels have generated research that supports broader implications of Piaget's cognitive theory in addition to providing useful information to educators. Doyle (1978) states it best, saying that research can tell us how we learn but not how to teach. Thus, Piaget's work should not be the only factor in determining educational goals. It should simply serve as a guide.

The literature suggests that science and math curricula may be enhanced in a variety of ways. Taylor (1996) suggests that a curriculum would be more productive if teaching goals, pedagogy, and evaluation were unified under one theory. Her conclusion is that Piaget's theory is the most comprehensive theory that can achieve a fully developmental curriculum. Ideally, education would want to do as Raven (1973, 1977) suggests: individually diagnose the existence of logical operations and then provide individual instructional approaches to deal with the acquisition difficulties. This seems to be a case of the ideal, and educators face many issues that make this solution seemingly impossible. Lawson (1975,1976) provides a more practical approach in suggesting that elementary and junior high math and science curricula provide students with more hands-on experiences and concrete problems, ensuring that more students entering the secondary level are prepared for formal thinking. He also suggests that these experiences and problems should be considered with the developmental level of the learners in mind. Lawson (1976) and Doyle (1978) contend that the curriculum should contain a wealth of materials, resources, and activities for the students in a sequence that progresses from the concrete to the abstract. Doyle further elaborates by stating:

> "a limited number of basic concepts should be studied from different perspectives and throughout numerous intensive experiences. These experiences should be centered around manipulative activities that are relevant to students who are at different levels of development. Introductory courses should emphasize the development of scientific knowledge rather than survey an array of facts and abstract models" (Doyle, 1978, p. 479).

A curricular commitment to the literature suggestions would certainly be the first step in fostering students' development to formal thought, but there are many suggestions from the literature concerning classroom practice as well. Almost all of the literature suggests that it is to the teacher's benefit to use Piagetian tasks or other measures to identify each student's cognitive abilities. Taylor (1996) suggests that this should be seen as a way to help students on their progression toward formal operations. Monk (1995) further supports this idea by suggesting that students must be provided with the opportunity to pass through stages of development. Lawson (1975) contends that many concepts in the secondary science curriculum are formal in nature and courses should be taught at the concrete and formal levels to identified students. Lawson does indicate that identification has its drawbacks, mainly because instruction must lure students to the formal level and strictly concrete instruction may stifle development. Herron (1975) is more optimistic in stating that understanding can be achieved for many concepts by concrete students, and ways exist to make many of these abstract concepts concrete in nature.

The literature agrees that the classroom must be hands-on and experiential in nature in order to meet the needs of a variety of different levels of cognitive development. For some educators, hands-on is equivalent to rolling out the ball in physical education. There is a vital component that makes these hands-on experiences a true pathway to a higher operational level. The vital component is *communication*. Taylor (1996) emphasizes that communication between students develops perspective. During students' dialogue, conflicts arise, in which students must confront the limitations of their understanding to resolve these conflicts. Herron (1975) extends this idea to cognitive stages by implying that *student discourse, justification of claims, and emphasis on making sense of observed phenomena lead to formal thought.*

Although Bliss (1995) argues against the concept of attaining formal thought in Piaget's sense, she does contend that there is justification for Piaget's theory with mental models. Mental models are an extension of concrete reasoning. These mental models are simply tools for thinking, providing examples and counter examples from which new mental frames of reference can be achieved. She contends that Piaget is speaking the same language simply from an epistemological point of view. Bliss (1995) cites work that suggests a fundamental ontological space where objects and events can be placed for evaluation. *Bliss (1995) sees mental models as a way of constructing understanding through organization and use of mental tools for thought. These mental tools are* *analogous to concrete experiences at the formal level. The idea of models may be an explanation for the discrepancies with Piaget's formal stage. Sub-stages may be levels of development where models are not fully developed, missing, or connections between models are absent.* Further research on mental models could provide new insights for the best approaches to content considered formal in nature, like proportional reasoning.

A final concern for classroom implications suggested by the literature is the role of the instructor. Most of the literature refers to the instructor as a coach or a guide, helping the student achieve higher levels of development. Another suggestion from Driver (1978) is that instructors must become their own diagnosticians. Driver contends that it is the instructor's responsibility to be aware of problems in specific content areas, understand the possible interpretations that students may have, and prescribe the appropriate treatments.

II. MODELING INSTRUCTION

The Modeling Method of Instruction in physics is a constructivist approach to physics instruction that incorporates the considerations discussed above for development of formal operational thinking. The Modeling Method was developed by Dr. Malcolm Wells (now deceased) at Marcos De Niza High School in Tempe, Arizona, from the modeling theory of physics instruction of Dr. David Hestenes at Arizona State University (Hestenes, 1987). It has been highly successful in correcting misconceptions in students' thought with concepts in physics that are considered to be formal in nature (Wells, Hestenes, & Swackhamer, 1995). The method requires students to confront misconceptions in their thinking by employing conceptual tools that enable the student to identify better alternatives. It uses a constructivist approach to develop qualitative and quantitative thinking tools (models), and it makes extensive use of student discourse. Modeling Instruction is an interactive engagement method (Hake, 1998a) with an explicit focus on developing conceptual and mathematical models of physical phenomena. I learned Modeling Instruction in a three-week intensive workshop in summer 1999.

The following description of Modeling Instruction addresses cognitive foundations of the method, the role of discourse in modeling, and its implications for teaching. The information for the summary was obtained from transcripts of a series of lectures given by Dr. Hestenes (online at <http://modeling.asu.edu/>).

The cognitive foundations of Modeling Instruction are based primarily on trying to make sense of our experiences. This is inherent in human nature. Hestenes indicates that people make sense of experiences by constructing mental models. These models are developed by recognizing patterns in one's experience and are captured by formulating representations of those experiences. The recognition of patterns is directly related to formulation of analogies that in turn lead to models representing structure in a system or process. Hestenes discusses that analogies are resources for domains that are familiar but they can also be applicable to domains that are not familiar. He indicates that the ability to map structures from one domain to another is analogous to Piaget's schemata.

Hestenes describes three ways of applying the use of analogies. The first is identifying analogies between different systems. The second is transferring the structure (model) into meaningful representations: diagrams, equations, and graphs. The final aspect consists of drawing analogies between theoretical domains. He notes that analogies are not provided for us. Instead, they are developed from recognition of patterns implicit in nature. His interpretation of cognitive processes primarily accounts for the constructing of models and the search for similarities between those structures.

Hestenes indicates that these mental structures or models are essential for evaluating experiences. The goal of Modeling Instruction is to make these structures (models) apparent to the student and then develop the models through actively engaging in discourse. Quality discourse can only be achieved if students are placed in situations where communication of models and evaluation of models are routine classroom practice. The key to effectively communicating models for evaluation lies in shared meaning within the classroom community. The Modeling Method of instruction uses whiteboarding as the platform for classroom discourse.

The effect of evaluating and justifying claims related to one's experiences is compounded by centering the Modeling Method around a constructivist approach. Hestenes argues that this is the key to conceptual change. A modeling approach places students in situations where they can recognize a better alternative for their explanations through use of models. It is not necessary to tell them that there is a better alternative. If they are given appropriate tools, they will recognize an alternative that yields conceptual understanding. The tools are specific skills related to the construction of models. The role of the instructor is primarily focused on developing tools necessary for the creation of models that describe, explain, predict, and control physical phenomena. Once models are developed, the instructor seeks to place students in situations where recognition of patterns emerges through application of model(s) in a wide variety of situations. Hestenes argues that this process develops highly functional members of society who are capable of formulating their own judgments and evaluating evidence that supports their claims. He contends that this is applicable not only in physics, but in all aspects of becoming a well-informed member of society.

III. TEACHING PROPORTIONAL REASONING

Piaget's concept of formal operational thought is often associated with one's ability to reason proportionally (Bar, 1987; Tourniaire & Pulos, 1985). "The attainment of proportional reasoning is considered a milestone in students' cognitive development" (Cramer & Post, 1993, p.404). The importance of this statement can be justified with proportional reasoning's association to Piaget's formal operational stage of thought. Many researchers in Piagetian and neo-Piagetian research use tasks that require the use of ratio and proportion to identify the formal operational stage (Roth & Milkent, 1991). *Proportional reasoning is the ability to compare ratios or make a statement of equality between two ratios* (Bar, 1987, Tourniaire & Pulos, 1985). Piaget described the development of proportional reasoning in three stages. First, students are not aware of ratio dependence and seek solutions by guessing. Second, students are aware of objective dependence. Students seek solutions by estimation and later calculation, but assume that the change in one quantity produces the same change in the other quantity. In the final stage, proportionality is discovered and applied to obtain correct solutions (Vollrath, 1986). A literature review by Tourniaire and Polos (1985) suggests that proportional reasoning is much more complex than Piaget thought. There are many cognitive and developmental factors affecting the ability to reason proportionally.

A. Cognitive and developmental factors affecting proportional reasoning

A study by Moore, Dixon, and Haines (1991) distinguished between intuitive and computational formal reasoning abilities with proportional reasoning tasks. The design of their study allowed them to establish fuzzy boundaries. These fuzzy boundaries identified degrees of membership to proportional reasoning while allowing for individual differences and a more appropriate description of the developmental sequence. Their findings were not consistent with Inhelder and Piaget's conclusion that proportional reasoning naturally arises from fully developed intuitive proportional reasoning. In addition, the results are inconsistent with a single path of development, which is often the focus of cognitive development research. This suggests that distinct categories are not necessarily incorrect, but further clarification of an individual's degree of membership within the fuzzy boundaries of proportional reasoning may be helpful in its development. If this approach identifies the possible alternative pathways for achieving proportional reasoning and accurately identifies students' degrees of membership, further research could produce information on appropriate educational treatments. Successful treatments could then be applied to make adjustments in pedagogy and curriculum design that facilitate development of proportional reasoning.

In addition to clarifying developmental aspects of proportional reasoning, many variables affect ability to reason proportionally. M-capacity, Field dependence-independence (FDI), and short-term storage space are terms for abilities associated with cognitive ability and formal operational thought. M-capacity is related to the number of schemes that can be evaluated at one time, while FDI is associated with the susceptibility to misleading information and choice of strategy. Tourniaire and Pulos (1985) suggest that there are relationships between M-capacity and ability to reason proportionally for specific types of proportion problems. They also cite a relationship between success on proportional reasoning tasks and field independent subjects. Niaz (1989) found a strong correlation between FDI and proportional reasoning and concluded that field dependent students capable of reasoning proportionally could be misled by altering field factors. Roth and Milkant (1991) suggest that FDI and M-capacity become insignificant when developmental level is considered. Although their data did not significantly demonstrate that short-term storage space is a contributing factor, it could be hypothesized from their findings. They suggest that lack of short-term storage space could be an indicator of student difficulty with developmental task instruction. A more complete understanding of these factors could lead to curriculum design that helps tailor instruction in proportional reasoning to the individual learner.

Another cognitive factor affecting proportional reasoning is prefrontal lobe activity. Kwon et. al. (2000) found that factors associated with prefrontal lobe activity such as inhibiting ability, planning ability, disembedding ability, and mental capacity, all correlate significantly with ability to reason proportionally. Their findings suggest that maturing prefrontal lobes play a role in proportional reasoning ability and can be used to predict responsiveness to instruction focused on improving proportional reasoning. Their suggestion is that proportional reasoning instruction will be most effective if it occurs after the plateau period of brain maturation. This could be support for Tourniaire and Polos's (1985) findings that proportional reasoning increases dramatically with age. These ideas indicate that the developmental level of the learner should be considered when making decisions concerning introduction and sequence of proportional reasoning within the school curriculum.

B. Contextual factors affecting proportional reasoning

In addition to cognitive and developmental aspects, many task-related factors affect proportional reasoning. Heller et al. (1989) indicate that factors such as problem format, the particular numbers used in problems, problem context, and even the problem(s) preceding a task, influence proportional reasoning. Problem context seems to be a focal point for a majority of the primary research in proportional reasoning.

Contextual factors are inherent to the variety of tasks associated with proportional reasoning. Heller et al. (1989) organize contextual factors into two categories. The first category is problem setting, which includes objects in the problem, the variables used to describe the objects in the problem, and the units of measurement for each variable. Problem setting is usually categorized as either familiar or unfamiliar depending on student experience. The second category is ratio type. Anamauah-Mensah et al. (1987) and Vollrath (1986) distinguish between direct ratio and indirect ratio type problems, although direct ratio aspects seem to predominate the literature. Lamon (1993) used four categories to describe direct ratio type problems. These categories include well-chunked measures, part-part-whole, associated sets, and stretchers and shrinkers. These ratio types can be further classified as integer and non-integer types. The following literature presents the complex nature of these variables and their effects on proportional reasoning.

Lawton (1993) suggests that problem setting, the presence of discrete or continuous quantities, and familiarity with the content are factors responsible for variability in performance on proportional reasoning tasks. Lawton states that "understanding of proportion concepts is relatively fragile and easily influenced by structural variations in the problem" (p.460). Lawton's study focused on the degree of similarity between objects in proportional reasoning tasks and whether or not the degree of similarity influenced the use of proportional reasoning. She found that the more distinct the items are from each other, the more readily subjects used proportional reasoning.

Saunders and Jusenathadas (1988) conducted research to identify if students' proportional reasoning abilities generalize across subject-matter domains. They found that even when students possess proportional reasoning abilities, they are unable to apply these strategies to unfamiliar content like abstract science concepts. They did contend that these findings are less significant for simple tasks rather than more complex problems. Singh (2000) found similar results with high achieving students on national exams. Singh found that high test scores were not indicative of students' proportional reasoning knowledge and skill when solving complex and unfamiliar proportional reasoning tasks. This was true even when tasks were within the student's zone of potential construction.

Heller et al. (1989) investigated ratio type and problem setting along with factors such as rational number skills, qualitative directional reasoning, and numerical proportional reasoning. They found that ratio type influenced both familiar and unfamiliar tasks, although the effect was more significant for the unfamiliar. Unfamiliar settings proved to be much more difficult, especially with complex ratio types. There was also a significant effect when comparing familiar to unfamiliar settings. Heller et al. also concluded that directional reasoning is helpful but not necessary for success on proportional reasoning tasks. Heller et al. recognized that this effect is more than likely due to the use of a memorized algorithm. The relationship between proportional reasoning and rational number skills is different than expected because rational number skills are not necessary for success on proportional reasoning skills but help assure proportional reasoning success.

Bar (1987) investigated ratio concepts in two domains. The research provided interesting findings on the types of ratios used and the role of variables in the problem setting. Bar found that as students move from simple to more complex ratio types, the number of correct responses decreases. Bar also concluded that the same ratio type problems were more difficult for the students than direct or inverse ratio problems due to an incomplete understanding of the variables. The effect of variable understanding was a contributing factor in one of the tasks in which the role of one variable was completely understood and the other variable was ignored, thus leading to incorrect responses even when it was established that the reasoning required was present in another task. Bar suggests that differences in field effects, discreet versus continuous quantities, and familiarity with the content are factors that can contribute to the findings. Bar concluded that in order for a transfer to occur across subject matter domains, a perceptual analogy between the domains must be present.

Moore, Dixon, and Hanes (1991) cite literature indicating that problems involving the same mathematical processes can vary greatly in difficulty. Reversing tasks proved to be more difficult for students and problems involving unknown final states were easier than those in which the final states were given. In addition, multiplication and division operations were more difficult than addition and subtraction, especially for tasks involving non-integer numbers.

Bar (1987) and Anamuah-Mensah (1987) indicate that the direct ratio more closely resembles familiar situations and its understanding precedes that of the inverse ratio. Vollrath (1986) contends that students tend to assume direct proportionality and have difficulty overcoming this assumption when asked to make predictions on proportional reasoning tasks that do not represent direct relationships between variables. This could be evidence for why students have problems with reasoning tasks that require both direct and indirect proportional reasoning. Anamuah-Mensah (1887) attempted to identify reasoning pathways for direct proportionality, indirect proportionality, and prerequisite concepts for students who used proportional reasoning with and without understanding. Anamuah-Mensah found that inverse proportionality was necessary for solving tasks requiring the use of prerequisite concepts and involving direct and indirect proportional reasoning. Students with inverse proportionality skills were able to recognize structural relations necessary for determining solutions, while those who lacked inverse proportionality skills did not. Subjects who did not use proportional reasoning with understanding became confused when attempting to determine whether direct or indirect proportional reasoning was applicable to the variables in the problem.

C. Diagnosis of cognitive processes

The literature suggests that many cognitive, developmental, and contextual factors are involved in attainment of proportional reasoning. Educators' awareness of these factors could be the impetus for change in how development of proportional reasoning is approached throughout the education of a student. Cognitive and developmental factors should drive the introduction and progression of proportional reasoning instruction, while the concept is developed through a systematic progression of contextual aspects. The proposition of semi-dense test items may be a diagnostic tool to help educators evaluate the cognitive processes involved in proportional reasoning. Behr & Lesh (1994) cite literature that suggests “cognitive diagnosis is necessary for the prescription of effective instruction in mathematics”(p.1). The concept of a semi-dense item centers around exact interpretation of the errors students make when they respond to diagnostic items. This stems from identifying the sequence of cognitive operations that produce item responses (Behr & Lesh, 1994). Behr and Lesh indicate that items are semi-dense if at least one cognitive operation can be identified from a response, each response can be interpreted by a specified cognitive operation, the cognitive operations discriminate one response from other responses, and every cognitive rule interprets at least one response to the item. Behr and Lesh suggest that these conditions are ideal for items but there may only be a few items that meet the suggested criteria. Their suggestion is that items should meet as many criteria as possible to aid in diagnosis for a particular task. This type of diagnostic evaluation could help establish a developmentally appropriate sequence of tasks that facilitate development of proportional reasoning.

D. Proportional reasoning and instructional obstacles

Traditional mathematics instruction seems to contradict a universal awareness of the factors affecting proportional reasoning. Proportional reasoning implies that it is a reasoning technique that should be used to evaluate proportionality concepts. Traditional math instruction seems to leave out the reasoning part of the equation. Guckin and Morrison (1991) indicate that traditional math instruction emphasizes routine algorithmic problem solving rather than development of a reasoning strategy. Their interviews with college freshman support this statement. Most college freshmen indicated the importance of mathematics, although their interpretation of mathematics was limited to a collection of formulas and algorithmic procedures that are memorized to obtain answers. These students expected to be shown an algorithm that was to be practiced and memorized. Niaz (1989) contends that since the focus of math instruction is on manipulative skills, many students can manipulate equations and pass exams, yet they cannot comprehend real-world everyday problems. Epstein (1998) claims that many educators are not aware that students tend to lose all sense of understanding and meaning in mathematics during elementary school. This problem more than likely compounds as students are forced to complete a math curriculum with the characteristics described above. Heller et al. (1989) argue that it is debatable if traditional math instruction would ever be able to prepare students to transfer math skills to unfamiliar contexts, like those associated with science.

Instructional problems are not only attributed to mathematics instruction. Science instruction also is at fault in the difficulty of students’ understanding of proportional reasoning. Harriet and Wallace (1999b) studied socio-cultural factors affecting use of proportional reasoning on physics tasks. They found that students were not only unaware that proportional reasoning could be used to solve physics tasks, but that they did not view the concept of ratios as a problem solving tool. The students believed that there were specified ways to solve physics tasks and that they were expected to conform to those methods. Harriet and Wallace also found that students indicated that physics textbooks and instructors did not encourage use of ratios or proportions and texts simply provided formulas that were to be memorized. This caused students to feel as though proportional reasoning was a last resort.[2]

Harriet and Wallace (1999a) completed comprehensive research on students’ problems, difficulties, and understanding of mathematical proportional reasoning in physics. The outcomes of

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[2] Harriet and Wallace indicated that students had problems understanding the given tasks and the lack of understanding words, terms, units, and concepts created fear and tension, subsequently causing students to lose confidence and discouraging them from further attempts at finding a solution. Other factors contributing to students’ lack of success were poor student understanding of physics concepts and measuring devices required to complete the problems. The students associated this with lack of questioning during physics instruction. They believed that society did not encourage students to ask questions. They felt that their role was to be seen rather than heard during classroom instruction.

their study indicate that students employ algorithms that they can not explain and consistently have difficulty translating physics tasks into equations, symbols, and relationships. Their conclusion was that students did not have adequate understanding of the mathematical processes to perform well on physics tasks involving proportional reasoning. Students were not aware that proportional reasoning patterns were supposed to be meaningful or understood. In addition, they frequently exhibited problems with physics content, language, and interpretation of unfamiliar problems. Harriet and Wallace indicate a need for students to be aware of the meaning behind proportional reasoning patterns and how they are derived. They also conclude that future success is limited by the absence of cognitive structure along with proportional reasoning skill. They suggest that physics instruction should focus on the use of mathematical language, problem solving strategy, and the language, symbols, and statements that students use to represent physics tasks. Finally, Harriet and Wallace suggest that "students must move beyond rules to be able to express scientific ideas and information in the language of mathematics" (p. 38). Their consensus is that if attention is paid to these problems and incorporated into physics instruction, then mathematics and physics learning can be strengthened.

Another area that Aldridge (1994) contends is essential for the development of mathematical and scientific reasoning is use of symbols. He argues that students do not understand these distinctions and that NCTM standards do not give adequate attention to this issue. The final aspects that Aldridge associates with lack of mathematical and scientific understanding are the processes of induction and logical deduction. Lack of attention to these issues can be attributed to memorization of prescribed formulas in physics and an algorithmic approach to teaching mathematics. Aldridge argues that math and science instruction is attempting to make the subjects more relevant to motivate students, although, the power of sudden insight and deep understanding are overlooked. Aldridge contends that insight and understanding are much more powerful experiences for sustained efforts to learn more.

E. Proportional reasoning and science

Heller et al. (1989) point out that proportionality is one of the most ancient and fundamental connections between math and science. The extensiveness of proportionality in scientific principles indicates that a deep robust understanding of proportionality is necessary for students to make sense of many scientific principles (Aldridge,1994). The profoundness of these statements implies that development of proportional reasoning is crucial to student success in science.

Krajcik and Haney (1987) cite literature suggesting that many of the abstract concepts in chemistry require formal thinking but there is evidence showing that students who take chemistry are operating at various stages of cognitive development. Many of these students are at the concrete operational or a transitional stage to formal thinking. Lawson (1975) found that concrete operational students are unable to develop an understanding of formal concepts, indicating that understanding of chemistry concepts may be inhibited by lack of proportional reasoning skill. Krajcik and Haney analyzed the American Chemical Society Exam and found that over 50% of the test involves tasks requiring proportional reasoning. This implies that *proportional reasoning is the primary reasoning construct required for success in chemistry and complete development is crucial for achieving understanding of the many formal concepts associated with the content.*

Akatugba and Wallace (1999) contend that *almost every concept in physics requires a proficient understanding of proportional reasoning, and students who are not capable will have difficulty mastering these concepts.* Lawson and Renner (1975) also associate this problem with biology concepts. Although primary research was not obtained associating proportional reasoning with success in these disciplines, it can be hypothesized from the chemistry data that similar results would be obtained by analyzing physics and biology aptitude measures.

F. Instructional implications

Considering all of the variables affecting proportional reasoning, its relationship to formal operational thinking, and necessity for scientific understanding, *it is crucial for math and science educators to be aware of instructional implications suggested by the research. Such awareness should lead to improved instruction and serve as an impetus for curriculum reform.* Efforts in these areas could lead students down a pathway that fosters formal operational thinking, which will eventually produce students that are more prepared to handle the formal concepts that are so prevalent in secondary science courses.

Tourniaire and Pulos (1985) indicate that proportional reasoning is not a unitary construct, and therefore that a linear teaching sequence is not adequate for this multi-faceted reasoning strategy. They argue that different instructional approaches may be necessary for different context variables and number structures, in addition to considering cognitive ability, timing and sequence. Singh (2000) identifies *assessment* as one of the major factors discouraging an outlook similar to that suggested by Tourniaire and Polos. Singh believes that *schools place more emphasis on manipulative skills and memorized procedures rather than reasoning because a narrow range of skills are typically assessed*. This approach raises test scores but leaves students with islands of superficial knowledge blinding them from seeing the richly-interconnected spaces that are crucial for mathematical knowledge and sound reasoning abilities. Singh suggests that instruction must be carefully varied to expose students to all aspects and variables involved in proportional reasoning. This ensures that students are not left with one particular interpretation of aspects involving ratio and proportion. Instruction must be oriented to students' intuitive understanding of ratio and proportion rather than solely focused on manipulative skills. Singh argues that a manipulative approach eliminates the possibility of students to become proportional thinkers because it is highly dependent on memory and is subject to deterioration.

Typically a manipulative approach involves employment of an algorithm to obtain solutions to problems. In proportional reasoning, the algorithm that is taught is the cross-multiply and divide algorithm, or typically referred to as cross-multiplication. Farrell and Farmer (1985) cite literature that suggests "teaching which stresses "cross-multiplication" actually inhibits the development of the understanding of proportion" (p.517). Akatugba and Wallace (1999) recognize that this algorithmic approach focuses on a skill to be mastered rather than treating proportions as a reasoning construct. They cite literature indicating that students can mimic these procedures but the absence of an underlying cognitive structure limits future success and understanding. They argue that students must be aware of proportional reasoning patterns and understand how the relations are derived in order to move beyond rules and procedures. This in turn allows students to understand and express information in the language of mathematics. This supports Saunders' and Jesunathadas' (1988) speculation that the algorithmic approach is the contributing factor for students' lack of success when applying proportional reasoning to unfamiliar content.

Evidence against an algorithmic approach is supported in a study by Moore, Dixon, and Haines (1991). They found that mapping relations from one domain onto another requires understanding of how variables function and an intuitive understanding of the computational task. If intuitive understanding is not present, then the computational scheme is based primarily on the memory available of mathematical operations. Krajcik and Haney (1987) identified a relationship between cognitive development, memorization, and proportional reasoning tasks associated with chemistry. They found that non-formal operational students typically try to memorize procedures, often becoming confused due to the absence of a proportional reasoning strategy. This in turn leads to a lack of success on proportional reasoning tasks that are prevalent in chemistry courses.

Roth and Milkent (1991) suggest that science and math educators should attempt to develop a curriculum that matches short-term storage space capabilities to the storage space demands of proportional reasoning tasks. This would aid in diagnosing transfer of proportional reasoning schema to new situations, as tasks become more complex through a range of problem contexts.

The literature concerning an algorithmic approach and its failure to develop proportionality as a reasoning strategy seems to suggest that math and science disciplines are highly dependant on one another for comprehensive development of this formal reasoning construct. Unification between these two disciplines could facilitate transferability of this formal cognitive process to the unfamiliar domains encountered in science. Akatugba and Wallace (1999) call for an integrated perspective that focuses on how students develop the ability to reason about complex proportional reasoning situations. They suggest that a constructivist approach to teaching proportionality can facilitate development of students' proportional reasoning skills, thus strengthening the bond between math and science. Akatugba and Wallace view constructivism as instruction that takes into consideration what the student already knows and emphasizes the active, constructive role of the learner in acquiring knowledge. The role of the instructor requires them to listen carefully to emergence of concepts through the student's voice and requires them to continually rethink their own perspectives.

Epstein (1998) describes an integrated math and science program aimed at reversing cognitive defects and enhancing sense of meaning and understanding through a constructivist approach. His methods are geared toward developing mathematical and scientific ideas necessary for developing scientific literacy through expression in the language of mathematics. His program is based on developing mathematical understanding through direct experience in scientific approaches. Much like constructivism, his program implements a guided inquiry approach that never tells the students what to do. The approach relies on active participation of the student by requiring them to hypothesize, allowing them to make mistakes and seek out blind alleys of approach, and learning from experience what approaches are successful. The role of the instructor is to use re-directive strategies such as questioning and reference to observations that facilitate the student's thought process, although never providing a correct answer. Instructors are patient with extensive wait times as well as not tolerating provision of answers from other students or themselves. This process facilitates the inductive nature of learning through trial and error, categorization, model development, and mental imagery that is consistent with direct experience. The learners in essence come to an understanding when they can formulate a mental picture that becomes a model for a solutions strategy. Throughout the whole curriculum there is not a trace of anything that is easily memorizable. These instructional methods are coupled with a journal that requires the students to put their learning into their own words. This platform serves as a tool for constant regulation and self-evaluation of the learning process. The program produced astonishing improvement in mathematical and scientific reasoning as well as attributed to greater success in first year chemistry courses.

Another research study on constructivism and proportional reasoning was conducted by Guckin and Morrison (1991). The study consisted of developing proportional reasoning through a computer based micro-world program. The micro-world is based on a computer language that allows students to explore concepts of ratio and proportion. Students developed proportional reasoning skills by discovering how to function in the micro-world through the logo language. The constructivist approach had a large effect on the experimental group in that all students receiving the treatment were successful on formal reasoning tasks associated with proportional reasoning after treatment.

While the constructivist programs mentioned above are exemplary examples of how constructivism can be successful at developing mathematical and scientific concepts, the commitment and cost of implementing such programs may be outside the means of most public education systems. Vollrath (1986) and Heller et al. (1989) make suggestions for improving math and science instruction in relation to proportional reasoning. Vollrath suggests that math instruction should be initiated through scientific experiments. These experiences would provide for a more constructivist approach to math concepts. These problems lead to conjectures that must be validated through mathematical or experimental justification. Heller et al. support this constructivist suggestion seeing value in developing the mathematics to deal with situations rather than look at the situations as an afterthought application. Heller et al. also suggest that if a unified approach is not available, then science instructors must not assume that math instruction has produced fully functional proportional thinkers. They contend that science instruction should begin easy, focusing on intuitive understanding of directional changes in proportional concepts before proceeding to quantitative examples. This would also be aided by familiarizing students with objects and units associated with the proportion task.

Other suggestions related to math and science instruction were found, and these suggestions could easily be integrated into a constructivist approach and may be necessary for a constructivist approach to be complete. Aldridge (1994) argues that the greatest need for a unification of math and science is for instruction to focus on using, distinguishing between, and understanding the units and symbols that are the foundation of the mathematical language. Akatugba and Wallace (1999) have similar suggestions but include that a strict focus on the language students use when speaking mathematical language is the essence of developing meaningful problem solving, especially on science related tasks. Lawton (1993) identifies a unit approach as being crucial to intuitive understanding. A constructivist approach would certainly provide a shift in focus for these suggestions because the constructivist approach focuses on the student's interpretation.

Other suggestions that would easily be incorporated into a constructivist approach focus on applications of approach and shift of student focus. Cramer and Post (1993) advise that instruction in proportional reasoning should start with familiar contexts and extend to unfamiliar. They also suggest that teachers should begin with more intuitive strategies and focus student attention to multiple strategies for any given problem. This helps emphasize learning concepts over procedures. Lammon (1993) suggests that instruction must focus on the ratio as a new entity from two distinct quantities. In addition to developing the ratio concept prior to proportional reasoning, Lammon points out that problems must be organized so that students can make comparisons. These comparisons would then allow students to focus on developing criteria to evaluate given situations. Finally, Saunders and Jesunathadas (1988) suggest that there needs to be a focus on problem identification and recognition: instruction must provide problems where students must identify the mathematical relationship presented by the variables and apply appropriate solution strategies. These suggestions could evolve through a constructivist approach and may be a determining factor for developing the ability to use strategies in unfamiliar situations.

The final literature suggestion to improve proportional reasoning abilities that would be facilitated by a constructivist approach is developing a hands-on approach, as discussed by Farrell & Farmer (1985), Krajcik & Haney (1987), Kwon et al. (2000), and Tournaire & Pulos (1985). Tournaire and Pulos indicate that hands-on experiences can strongly influence some learners although they may not have any effect for a portion of the population. Kwon et al. states the importance of hands-on experiences and manipulatives in that they provide unexpected events triggering bursts of cognitive arousal, while Farrell and Farmer found that students are more likely to succeed when provided with concrete hands-on experiences. These reasons could be indicative of Krajcik and Haney’s conclusion that non-formal operational students benefit from a more concrete approach including the use of manipulatives through hands-on experiences. This supports a problem based constructivist approach that initiates inquiry through experience.

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ADDENDUM

Dr. Richard Hake read an early draft of this paper and commented on it as follows: "You may be unaware of Baxter & Junker (2001), and the pioneering Piaget-related work of physicists Arons and Karplus (Fuller 2002)." Thus I include those references.

REFERENCES

Aldridge, Bill. (1994). The role of mathematics in science education. Focus on Learning Problems in Mathematics, 16(5), 36-44.

Akatugba, Ayo Harriet & Wallace, John. (1999a). Mathematical dimensions of students’ use of proportional reasoning in high school physics. School Science and Mathematics,99(1), 31-39.

Akatugba, Ayo H. & John Wallace. (1999b). Sociocultural influences on physics students’ use of proportional reasoning in a non-western country. Journal of Research in Science Teaching, 36(3), 305-320.

Anamuah-Mensah, Jophus, Erickson, Gaalen, & Gaskell, Jim. (1987). Development and validation of a path analytic model of students' performance in chemistry. Journal of Research in Science Teaching, 24(8), 723-738.

Association for Supervision and Curriculum Development (ASCD). (1997). International math and science study calls for depth, not breadth. Education Update.39(1), 1-8. Numerous TIMSS-related publications are online in pdf format at <http://timss.bc.edu/>.

Bar, Varda. (1987) Comparison of the development of ratio concepts in two domains. Science Education, 71(4), 599-613.

Bart, William M., Post, Thomas, Behr, Merlyn, & Lesh, Richard. (1994). A diagnostic analysis of a proportional reasoning test item: An introduction to the properties of a semi-dense item. Focus on Learning Problems in Mathematics, 16(3), 1-11.

Baxter, G.P. & B. Junker (2001). "Designing Cognitive Developmental Assessments: A Case Study in Proportional Reasoning," Paper presented at the annual meeting of the National Council for Measurement in Education, Seattle, Washington, April 2001; online at <http://www.stat.cmu.edu/%7Ebrian/bjtrs.html>.

Bliss, Joan.(1995). Piaget and after: The case of learning science. Studies in Science Education, 51, 139-172.

Cramer, Kathleen & Post, Thomas. (1993). Proportional reasoning. Mathematics Teacher, 86(5),404-407.

Doyle, Jerry J. & Lunetta, Vincent N. (1978). Piaget in perspective. School of Science and Mathematics. 78(6), 475-480.

Driver, Rosalind. (1978). When is a stage not a stage? A critique of Piaget's theory of cognitive development and its application to science education. Educational Research, 21(1), 54-61.

Epstein, Jerome. (1998) Cognitive development in an integrated mathematics and science program. Journal of College Science Teaching, 27(3), 194-201.

Friedel, Arthur W., Gable, Dorothy L., & Samuel, John. (1990). Using analogs for chemistry problem solving: does it increase understanding? School Science and Mathematics, 90(8), 674-682.

Farrell, Margaret A. & Farmer, Walter A. (1985). Adolecents' performance on a sequence of proportional reasoning tasks. Journal of Research in Science Teaching, 22(6), 503-518.

Fuller , R. G. ed. (2002). "A Love of Discovery: Science Education - The Second Career of Robert Karplus," Kluwer/Plenum.

Guckin, Alice Mae & Morrison, Dwight. (1991). Math logo: A project to develop proportional reasoning in college freshman. School Science and Mathematics, 91(2), 77-81.

Hake, Richard R. (1998a). "Interactive-engagement vs traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses," *American Journal of Physics* **66**, 64-74; online as ref. 24 at <http://www.physics.indiana.edu/~hake>.

Hake, Richard R. (1998b). "Interactive-engagement methods in introductory mechanics courses," online as ref. 25 at <http://www.physics.indiana.edu/~hake>, a crucial companion paper to Hake (1998a): average pre/post test scores, standard deviations, instructional methods, materials used, institutions, and instructors for each of the survey courses of Hake (1998a) are tabulated and referenced. In addition the paper includes: (a) case histories for the seven IE courses of Hake (1998a) whose effectiveness as gauged by pre-to-post test gains was close to those of T courses, (b) advice for implementing IE methods, and (c) suggestions for further research.

Heller, Patricia M., Ahlgren, Andrew, Post, Thomas, Behr, Merlyn, & Lesh, Richard. (1989) Proportional reasoning: the effect of two content variables, rate type, and problem setting. Journal of Research in Science Teaching, 26(3), 205-220.

Herron, J. Dudley. (1975). Piaget for chemists. Journal of Chemical Education, 52(2)146-150.

Hestenes, David (1987). Toward a Modeling Theory of Physics Instruction, American Journal of Physics 55, 440-454. Online in pdf at <http://modeling.asu.edu/R&E/Research.html>.

Hestenes, David. (1997). Lectures on Modeling Instruction in High School Physics [online]. Available: <http://modeling.asu.edu/modeling-HS.html>.
Kolodiy, George Oleh. (1977). Cognitive development and science teaching. Journal of Research in Science Teaching, 14(1), 21-26.
Krajcik, Joseph S. & Haney, Richard E. (1987). Proportional reasoning and achievement in high school chemistry. School Science and Mathematics, 87(1), 25-32.
Kwon, Yong-Ju, Lawson, Anton E., Chung, Wan-Ho, & Kim, Young-Shin. (2000). Effect on development of proportional reasoning skill of physical experience and cognitive abilities associated with prefrontal lobe activity. Journal of Research in Science Teaching, 37(10), 1171-1182.
Lamon, Susan J. (1993). Ratio and proportion: Connecting content and children's thinking. Journal for Research in Mathematics Education, 24(1), 41-61.
Lawson, Anton E. & Renner, John W. (1975). Relations of science subject matter and developmental levels of learners. Journal of Research in Science Teaching. 12(4), 347-358.
Lawson, Anton E. & Wollman, Warren T. (1976). Encouraging the transition from concrete to formal cognitive functioning-an experiment. Journal of Research in Science Teaching. 13(5), 413-430.
Lawton, Carol A. (1993). Contextual factors affecting errors in proportional reasoning. Journal for Research in Mathematics Education, 24(5), 460-466.
Monk, Martin. (1995). On the identification of principles in science that might inform research into students' beliefs about natural phenomena. International Journal of Science Education, 17(5), 565-573.
Niaz, Mansoor. Translation of algebraic equations and its relation to formal operational reasoning. (1989). Journal of Research in Science Teaching, 26(9), 785-793.
Moore, Colleen F., Dixon, James A., & Haines, Beth A. (1991). Components of understanding in proportional reasoning: A fuzzy set representation of developmental progressions. Child Development, 62(3), 441-459.
Raven, Ronald J. (1973) The development of a test of Piaget's logical operations. Science Education. 57(3), 377-385.
Raven, Ronald J. The development of a test of Piaget's operative comprehension. Science Education, 61(3), 271-278.
Roth, Wolff-Michael & Milkent, Marlene M. (1991) Factors in the development of proportional reasoning strategies by concrete operational college students. Journal of Research in Science Teaching, 28(6), 533-566.
Saunders, Walter L. & Jesunathadas, Joseph. (1988) The effect of task content upon proportional reasoning. Journal of Research in Science Teaching, 25(1), 59-67.
Singh, Parmjit. (2000). Understanding the concepts of proportion and ration among grade nine students in Malaysia. International Journal of Mathematical Education in Science and Technology, 31(4), 579-599.
Taylor, Janet B. (1996). Piagetian perspectives on understanding children's understanding. Childhood Education, 75(5), 258-259.
Tourniaire, Francoise & Pulos, Steven. (1985). Proportional reasoning: A review of the literature. Educational Studies in Mathematics, 16(2), 181-204.
Vollrath, Hans-Joachim. (1986) Search strategies as indicators of functional thinking. Educational Studies in Mathematics, 17(4), 387-400.
Wells, Malcolm, Hestenes, David, & Swackhamer, Gregg (1995). A Modeling Method for High School Physics Instruction, American Journal of Physics 63, 606-619. Online in pdf format at <http://modeling.asu.edu/modeling-HS.html>. See also "Findings of the Modeling Workshop Project, 1994-2000", online in pdf format at <http://modeling.asu.edu/R&E/Research.html>.

APPENDIX: Definitions

Concrete operational. Capable of processing information that can be physically manipulated and experienced.
Constructivist. Refers to the philosophy that supports the process of a student actively constructing knowledge in personally meaningful ways.
Disequilibrium. When a child encounters a situation that s/he has never experienced before, and/or his/her old ways of thinking are challenged, the child enters into a state of disequilibrium, or a state of intellectual conflict.
Formal operational. "Meaning is given to these concepts not through the senses but through imagination or through logical relationships within the system." (Lawson, 1975,p.348)
Interactive engagement methods. Operationally defined as "those designed at least in part to promote conceptual understanding through interactive engagement of students in heads-on (always) and hand-on (usually) activities which yield immediate feedback through discussion with peers and/or instructors", as opposed to "relying primarily on passive-student lectures, recipe labs, and algorithmic-problem exams" (Hake, 1998a,b).
Manipulative approach. The manipulations of numbers and variables through a specified process or set of rules. Usually used in conjunction with an algorithm and differing from a hands-on approach where objects, often referred to as manipulatives, are used to make the experience more concrete in nature.
Mathematical modeling. The systematic development of mathematical reasoning abilities through recognition of patterns between numerical data, graphical relationships, equations, and evaluation techniques that require use of mathematics as a language to establish relationships.
Metacognitive. Thinking about thinking.
Reasoning pattern. An identifiable and reproducible thought process directed at a type of task.
Rote. Learning strictly through abstract memorization.
Solved. Giving the right answer justified by the right interpretation.
Traditionalism. The type of learning philosophy that has been the norm for 100 years.
Whiteboarding. The process of using hand held marker boards to present solutions that are to be verbally justified by the presenter in addition to serving as a platform for classroom discourse.