Effect of Modeling Instruction on Development of Proportional Reasoning I: an empirical study of high school freshmen

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revised Aug. 28, 2003[1]

Abstract

Proportional reasoning is the ability to compare ratios or the ability to make statements of equality between ratios. Jean Piaget considers the ability to reason proportionally to be a primary indicator of formal operational thought, and this stage is viewed as the highest level of cognitive development. Proportional reasoning is taught primarily in mathematics courses, but student success in secondary science is highly dependent on proportional reasoning ability.

The focus of this empirical study was to compare traditional math instruction to instruction that is consistent with the Modeling Method of physics instruction and findings from proportional reasoning research. Twenty-one ninth grade students in the control group received traditional math instruction while 21 students received the treatment. A proportional reasoning test instrument representing a wide variety of contextual factors was developed from items found in the literature. An independent two-tail t-test was completed with the pretest and posttest data for both groups as well as between the net gain from pretest to posttest for each group. The treatment proved to be highly significant, $p<.01$, indicating that the treatment developed the necessary reasoning required for greater success on the proportional reasoning instrument. The success of the treatment suggests a viable model for mathematics instruction that could lead to greater success in secondary science as well as facilitate formal operational thought.

I. INTRODUCTION

The Third International Math and Science Study (TIMMS) produced startling data on U.S. students' achievement in math and science. Results showed that 8th grade US students rank below the international average in math and just above in science (ASCD, 1997), and 12th grade physics students rank lowest in the world. I am a teacher of chemistry and physics at a high school where the majority of students taking these subjects are typically in the top 25%-50% of their graduating class, but I have observed evidence that *many students lack not only the skills necessary for success in upper level science courses but more importantly the foundation of reasoning necessary to apply those skills under appropriate conditions.* I can only imagine how poorly students who do not attempt these courses would perform. This observational evidence makes it easy to understand the overall student population's lack of success on international studies.

In addition, these observations have led me to believe that the current curriculum design and methods of instruction used in science and mathematics courses are not appropriate for developing formal thought processes. Education should produce students who not only have a high level of literacy but also are capable of reasoning at the formal operational level. (Appendix A is a list of definitions for this and other technical terms.)

It would benefit student achievement on international studies and in post-secondary education, as well as develop highly functioning contributors to society, if instructional methods and curriculum design developed the student's mind to operate at the formal level. In my office, I keep a quote from the TIMSS report that states "No single vision for mathematics and science education exists in the United States" (ASCD, 1997, p. 1). It is my belief that educators must take this quote to heart, because *there is no better way of developing higher levels of thought processes than through math and science. Only through a unified approach can math and science facilitate development of formal reasoning for all students.* It is not that science instructors blame the math instructors and vice versa; rather, it is the lack of this unified vision that stifles the process. *Many aspects of science require use of math skills, and many math skills could be developed from a scientific approach. A curriculum that encompasses a unified vision for math and science may be the solution to producing greater numbers of students who operate on the formal operational level.*

The current educational system doesn't attempt to assure cognitive development to the formal level. Students can simply slip through the cracks without being accountable for mastering a variety of skills and reasoning techniques. The implementation of a standards and benchmarks approach to curriculum design is attempting to alleviate this issue, but its current state simply mirrors pre-existing curriculum design. Even with a standards and benchmark approach, math and science exist as separate entities within educational institutions. *This problem occurs for many reasons: the research to support a unified effort is not reaching educators or it is ignored; educators are not provided with time to collaborate; current assessment methods do not reveal enough about individual students' cognitive abilities; appropriate learning tasks are not matched with cognitive development; longitudinal sequencing does not facilitate mastery of concepts and skills over time; and teacher training in assessing individual students and appropriate instruction is lacking.*

Despite these problems, efforts exist that address these issues. Math and science programs do exist that provide professional development in appropriate methods aimed at developing formal operational thought. One is the Modeling Method of Instruction in high school physics, developed by Dr. Malcolm Wells (now deceased) at Marcos De Niza High School in Tempe, Arizona, from the modeling theory of physics instruction of Dr. David Hestenes at Arizona State University (Hestenes, 1987). Modeling Instruction has been highly successful in correcting misconceptions in students' thought with concepts in physics that are considered to be formal in nature (Wells, Hestenes, & Swackhamer, 1995). The method requires students to confront misconceptions in their thinking by employing conceptual tools that enable students to identify better alternatives. It uses a constructivist approach to develop qualitative and quantitative thinking tools (models), and it makes extensive use of student discourse. Modeling Instruction is an interactive engagement method (Hake, 1998a) with an explicit focus on developing conceptual and mathematical models of physical phenomena. I learned Modeling Instruction in a three-week intensive workshop in summer 1999.

Some of these professional development programs, such as Modeling Instruction, have been designed with cognitive development in mind, although most of them focus on a single course, content area, or range of grade levels. *In order to develop students' cognitive abilities fully, a comprehensive program in math and science must be in place throughout the duration of students' education. This program must ensure appropriate assessment of student ability, systematically develop cognitive ability with a wide variety of useful skills, and require application of those skills until mastery has taken place. This vision will not be achieved until practitioners are fully educated in cognitive development, assessment, and research.*

As a step in this direction, I attempt in this empirical study to identify an instructional model for proportional reasoning that helps facilitate formal operational thinking in addition to meeting the needs of math and science instruction. Proportional reasoning is the ability to compare ratios or the ability to make statements of equality between ratios. Jean Piaget considers the ability to reason proportionally to be a primary indicator of formal operational thought, and this stage is viewed as the highest level of cognitive development (Tourniaire & Pulos, 1975). Proportional reasoning is taught primarily in mathematics courses, but student success in secondary science is highly dependent on proportional reasoning ability. For example, Krajcik and Haney (1987) analyzed the American Chemical Society Exam and found that over 50% of the test involved tasks requiring proportional reasoning. This implies that *proportional reasoning is the primary reasoning construct required for success in chemistry* and complete development is crucial for achieving understanding of the many formal concepts associated with the content. Akatugba and Wallace (1999) contend that *almost every concept in physics requires a proficient understanding of proportional reasoning,* and students who are not capable will have difficulty mastering these concepts.

For those who wish to learn deeply, I provide theoretical background on research in scientific reasoning skills, teaching of proportional reasoning, and the Modeling Method of physics instruction (Wells et al, 1995; Hestenes, 1997) in an accompanying article (McLaughlin, 2003), obtainable at <http://modeling.asu.edu/modeling-HS.html> or by e-mailing me.

II. THE INSTRUCTIONAL STUDY

The purpose of this study was to determine if strategies used in Modeling Instruction, combined with findings from research in proportional reasoning, are more successful at developing proportional reasoning than traditional methods used in mathematics instruction.

A. Sample

Forty-two freshmen (9th grade) Math I (first year algebra) students participated in this study. They came from a population of approximately 90 freshman students enrolled in Math I. The study was conducted in Norwalk, Iowa, a homogeneous suburban community with a population of about 6000 people.

The experimental group consisted of a single Math I class that had no identified special needs students. The class met from 1:45 to 2:30 PM each day throughout the treatment; it consisted of eleven males and females. The experimental group received instruction that was consistent with the Modeling Method of teaching physics, and it considered implications for instruction from proportional reasoning research.

The control group consisted of eleven females and ten males randomly selected from the three Math I classes not receiving the treatment. The control groups' instruction time ranged from 12:05 to 1:40 PM or 2:35 to 3:20 PM. These students were randomly selected, to ensure the same numbers of each gender and to eliminate the possibility of comparing the experimental group to a single class that was specified for inclusion with a high number of identified special needs students. The control group received instruction consistent with traditional math instruction.

B. Experimental design

The study began on January 21, 2002 and concluded on February 4, 2002. The study began with a baseline pretest developed from example proportional reasoning tasks that I discovered in the literature. The proportional reasoning tasks included qualitative and quantitative measures, including a variety of context variables, integer and non-integer tasks, and familiar and unfamiliar situations. This test was administered after students had been instructed traditionally in ratios and rates, and it was given to all students receiving Math I instruction. Appendix B is a revision of the instrument.

Two weeks of instruction were allotted for completion of a proportional reasoning unit. The instructor for all Math I students up to that time in the year taught the control groups in a manner that is consistent with traditional math instruction. The experimental group received instruction from Mr. Shannon McLaughlin, who uses Modeling Instruction in high school physics and is familiar with research on proportional reasoning.

The experimental group's instruction was consistent with a constructivist approach and modeling methods, and it incorporated suggestions in the literature for improving proportional reasoning. Students were never prescribed a failsafe algorithm to obtain solutions. Instead, *mathematical modeling was done: mathematical reasoning abilities were systematically developed through recognition of patterns between numerical data, graphical relationships, equations, and evaluation techniques that require use of mathematics as a language to establish relationships.* Students were given tasks that required use of prior ratio knowledge and facilitated recognition and development of patterns that are necessary for success on proportional reasoning tasks. The experimental group's instruction included fewer problems but required students to show all steps in the process used to obtain a solution, written justification, and oral presentation followed by verbal justification during student discourse. Instruction included hands-on problems in addition to tasks requiring familiarization with scientific concepts requiring application of proportional reasoning.

The study concluded with completion of the same proportional reasoning instrument used for the baseline pretest, and the experimental group completed a written evaluation of the methods for qualitative feedback.

C. Details of experiment

Instructional methods to develop specific models necessary for understanding in physics were used, as well as suggestions taken from the literature on instructing proportional reasoning. The following description of the activities and instruction provide a clearer picture of how the study was conducted over a two week period and may serve as an initial starting point for further study concerning development of structure in students' proportional reasoning capabilities. Appendix C is most in-class assignments.

It is important to clarify that many of the instructional methods were new to students. The instructor had to teach the process of whiteboarding to students as well as attempt to stimulate discussion in a class where they traditionally are focused only on a numerical output; that proved difficult. In addition, students were not accustomed to not being prescribed an algorithm, having to show all steps of their work including units, and clarifying or justifying their solutions. (A description of the modeling cycle in physics instruction and clarification of the term whiteboarding can be found on the modeling web site for physics instruction, <http://modeling.asu.edu/modeling-HS.html>.)

Prior to the study, the students had concluded a unit on rates through traditional math instruction. It was evident that many students had trouble with the concept of rates at the start of the study; and some students had difficulty with the idea of fractions themselves. It is important to note that this study is not attempting to point the finger at math instructors; *it is inherently focused on what can be done in math instruction to further develop math concepts so that students have well developed models of formal thinking operations*, to prepare them for advanced math and science concepts.

The first day of the study consisted of a pretest and a problem that attempted to develop the concept of equivalent ratios. A simple problem was provided, the students were given work time, and group solutions were placed on whiteboards. The example problem was similar to this: If a grocer sells three apples to every four oranges, how many apples would the grocer expect to sell if x number of oranges were sold? Students were also required to solve the problem given the number of oranges sold. This was done for a variety of different scenarios including large and small numerical values but only *integer* values. In the concluding whiteboarding session, different groups were required to provide the different solutions, and facilitated discussion focused on the aspect of equivalence as well as the various methods of reasoning used to produce each solution.

The second problem set consisted of more complex problems using the idea of equivalence. An example is: A building company has been requested to build between 35 and 45 apartments. Each time they build three 1-bedroom apartments they must build four two-bedroom apartments and one three-bedroom apartment. What total number of apartments must they build and how many of each type will there be? The idea that there was more than one possible solution seemed to amaze students. Solutions ranged from drawing columns of boxes in sets of 3, 4, and 1 until they hit a range of 35 to 45, to doing complex numerical guess and checks. Other problems consisted of dividing pizza evenly amongst boys and girls as well as calculating unknown sides of boxes that were proportional to each other.

The third assignment consisted of solving even more complex problems that involved integer and non-integer values. A unique requirement of the assignment was that students were required to complete the following in order while doing the assignment.

1) Make a prediction for the missing value of more, less, or equal for each problem.
2) Determine a solution to the problem.
3) Explain your solution or show your work; put labels on your numbers and show all steps.
4) Prove your solution by setting up ratios and comparing them to each other.

A variety of problems were given, but all employed some significant suggestions from the literature. All problems were word problems, and only a small number of problems were given to ensure that the focus of the assignment was on meeting the above requirements. Meeting these requirements provided a wealth of discussion during the whiteboarding session.

The fourth assignment consisted of a drawing on a piece of graph paper. The drawing was of a triangle with a base of four and a height of five boxes or units. The triangle was extended to produce 4 proportionate triangles within the largest triangle provided. A variety of questions and application questions accompanied the graph paper. How many triangles do you see? Explain in words how you would describe the triangle to someone who could not see it. How are the base and height of the triangle related to each other? Use the idea of equivalent ratios to prove whether or not all triangles are similar. Can you predict the base and height of a fifth and sixth triangle? Predict the height of a triangle with a base of 2 units; how about 3.2 units? Other non-integer problems were posed, and the following application problem. Using what you have learned from this worksheet, devise a procedure for determining the height of a building in feet and inches. If the building's shadow is 25 ft. long, and your shadow is 3’6”, what is the height of the building?

These four assignments took 7 days. The remaining three days were spent doing hands-on applications of the idea of equivalent ratios and proportional reasoning. The primary activities were using marked beans (tagged deer) and unmarked beans (the rest of the deer population) to predict deer population in a forest. The other activities consisted of graphing a linear relationship and using a torque balance, to get at the idea of an inverse proportion. Although there was limited time to deploy these activities and applications, many connections were made between where and when proportions are used, how proportions are related to linear graphs, and how an inverse proportion is different from a direct proportion.

D. Quantitative assessment

Student scores on the pretest served as a baseline, and posttest scores were used as the basis for statistical analysis of this study. An independent t test with an alpha level set at .05 was used to determine significance of the study; and students' normalized gains were calculated.

Solutions to the proportional reasoning task were categorized according to the following scoring system.

0 = incorrect response to the task
1 = correct response by guessing or with no justification through showing of work or explanation
2 = correct response with one component of explanation or manipulations that occur by chance
3 = correct response with a prescribed algorithm or correct solution with incomplete explanation
4 = correct solution or the correct response along with the correct justification (interpretation)

III. RESULTS

A. Descriptive data

All statistical data were computed using Microsoft® Excel 95, copyright© 1995 from Microsoft® Corporation.

Table 1 displays all data for pretest and posttest scores for the control and experimental group on the proportional reasoning instrument. The difference column indicates the net gain or loss on the instrument, while the bottom row indicates the mean for each column. A maximum score of 48 was possible for correct solutions on the proportional reasoning instrument.

Table 1: Pretest and Posttest Data for Control and Experimental Groups								
Control ID#	C Pre-Test	C Post-Test	Difference		Experimental ID#	E Pre-Test	E Post-Test	Difference
cF03	22	24	2.00		eF43	25	24	-1
cF37	14	6	-8.00		eF96	28	33	5
cF117	10	12	2.00		eF88	22	22	0
cF111	18	18	0.00		eF83	13	21	8
cF04	21	24	3.00		eF79	31	37	6
cF112	24	26	2.00		eF70	25	35	10
cF119	8	11	3.00		eF65	26	35	9
cF86	23	19	-4.00		eF98	21	20	-1
cF130	11	11	0.00		eF08	16	29	13
cF107	37	34	-3.00		eF38	18	27	9
cF12	9	16	7.00		eF34	28	34	6
cM 129	13	11	-2.00		eM 25	16	28	12
cM 39	13	9	-4.00		eM 26	33	32	-1
cM 02	32	31	-1.00		eM 62	15	40	25
cM 94	29	28	-1.00		eM 75	26	42	16
cM 61	12	14	2.00		eM 100	8	16	8
cM 67	31	32	1.00		eM 102	17	23	6
cM 42	19	29	10.00		eM 41	21	39	18
cM 105	13	18	5.00		eM 27	23	21	-2
cM 73	18	13	-5.00		eM 19	6	12	6
cM 33	7	21	14.00		eM 113	15	32	17
Means	18.3	19.4	1.1			20.6	28.7	8.0

B. Inferential statistics

Table 2 displays results of an independent two-tail t-test for the control group's pretest and posttest scores.

Table 2: Control Pretest vs Posttest		
t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	Variable 2
Mean	18.28571	19.38095
Variance	73.71429	70.04762
Observations	21	21
Hypothesized Mean	0	
df	40	
t Stat	-0.4186	
P(T<=t) one-tail	0.338875	
t Critical one-tail	1.683852	
P(T<=t) two-tail	0.67775	
t Critical two-tail	2.021075	

Results in table 2 ($p=.68$) indicate no significant effect on the proportional reasoning instrument for traditional math instruction, $p>.05$. The chance of a type one error (Section IV) is very unlikely based on literature implications concerning traditional math instruction.

Table 3 displays the results of an independent two-tail t-test for the experimental group's pretest and posttest scores.

Table 3: Experimental Pretest vs Posttest		
t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	Variable 2
Mean	20.61905	28.66667
Variance	51.54762	68.23333
Observations	21	21
Hypothesized Mean I	0	
df	39	
t Stat	-3.30964	
P(T<=t) one-tail	0.000853	
t Critical one-tail	1.684875	
P(T<=t) two-tail	0.001706	
t Critical two-tail	2.022689	

Results in Table 3 ($p<.002$) indicate a highly significant effect on the proportional reasoning instrument for the experimental group. The chance of a type II error (Section IV) seems improbable. Proven modeling methodology along with use of instructional considerations in the literature probably enhanced proportional reasoning abilities.

Table 4 shows results of an independent two-tail t-test for the difference between pretest and posttest scores for the control and experimental groups.

Table 4: Control vs Experimental Net Gain		
t-Test: Two-Sample Assuming Unequal Variances		
	Variable 1	Variable 2
Mean	19.38095	28.66667
Variance	70.04762	68.23333
Observations	21	21
Hypothesized Mean D	0	
df	40	
t Stat	-3.61863	
P(T<=t) one-tail	0.000411	
t Critical one-tail	1.683852	
P(T<=t) two-tail	0.000822	
t Critical two-tail	2.021075	

Results in Table 4 (p=.0008) indicate a highly significant difference between net gains on the proportional reasoning instrument for the control and experimental groups, p<.001. The chance of a type II error is unlikely, considering that every aspect concerning instructional implications can be accounted for, in addition to using proven methods of instruction.

Another way of analyzing the data that is particularly useful for determining student learning is to calculate the normalized gain. Since a maximum score of 48 was possible for correct solutions on the proportional reasoning instrument, one may calculate:

<g> (test group) = (<post> - <pre>)/ (48 - <pre>
= (28.7 - 20.6) / (48 - 20.6) = 0.295

<g> (control group) = (<post> - <pre>)/ (48 - <pre>
= (19.4 - 18.3) / (48 - 18.3) = 0.037

which can be compared with normalized gains for interactive engagement <<g>>IE = 0.48 ±0.14 (std dev) and traditional introductory physics instruction <<g>>T = 0.23 ± 0.04 (std dev) (Hake 1998a,b).[2]

C. Commentary

Although the gains using this two week plan were significantly better than the control group, it is important to note that the experimenter's impression of the students' proportional reasoning capabilities was significantly below what it should be for students in a first year algebra course, and that many students would encounter significant problems in an advanced science course if further development was not ensured through a more thorough approach to proportional reasoning. As stated in the literature, proportional reasoning develops over a long period of time, typically ranging from months to years; therefore more action research is needed in the practical application of the large body of literature that concerns primarily the stages of development and the ages when these stages are present in students.

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[2] A complementary way to analyze data is by Cohen's "effect size" d. One way to calculate it is to find the normalized gain g for each individual student, the standard deviations sd(t) and sd(c), and g(ave-t) and g (ave-c) for the test and control groups, where g (ave) is the average of the individual student normalized gains. Then

d = [g(ave-t) - g (ave-c)] / [sd(t)^2 + sd(c)^2]^0.5.

For details and references on "d" and Cohen's rules of thumb regarding their significance, see Hake (2002).

D. Nurturant results

Although this study produced significant statistical results, they might not be generalizable to a larger population of students. Several variables working for and against the design of this study were out of the experimenter's control; they are discussed in Section IV. But even if the study were to be shown statistically insignificant in a more controlled experimental setting, several nurturant results became evident throughout the study.

The term nurturant is a psychological term that is derived from the nature vs. nurture paradigm. Does nature or the nurturing of a child have more to do with their development? In a teaching sense, instruction has nurturant effects. For instance, cooperative learning has nurturant effects in the sense that it provides students with roles, individual accountability, a task that can't be completed without teamwork, and it helps the students work on a specific social skill during the lesson. These aspects are separate from the content being taught.

The most significant nurturant effects of this study involve the students. The constructivist approach coupled with presentation, defense, and justification of claims created a more student centered learning environment. On post-instruction student evaluations, the experimental group had many positive comments about this type of instruction. Most students wrote that *this approach required them to think harder about the problems, providing a more intellectually stimulating environment.* Many students indicated that there was value in showing how solutions were determined and in the diversity of approaches used to determine solutions. Students indicated that these methods would generate more interest in mathematics and produce better understanding of concepts. It is also evident from post-instruction evaluations that students valued hands-on experiences, indicating that they are more intellectually stimulating and produce a better learning environment.

In relation to a better learning environment, the instructor's observations indicate that more opportunities for learning take place. (This is observed both in this situation and also in the instructor's use of the Modeling Method in physics.) Because problems were presented without an algorithm, students seemed to take more ownership in the problem solving process. Because students were in disequilibrium during the problem solving process, they spent more time focusing on reasoning to support their solutions. This made communication vital to success. Students spent time showing how they obtained solutions for problems and comparing their solutions to other members in their group. The student interaction provided time for students to teach one another. This is of value because students who have struggled through a problem will provide an understandable explanation to a student whose understanding is lacking. When the process is new to the students offering explanations, they will not assume that aspects of the problem solving process are understood. This type of atmosphere required students to be more accountable and developed the sense of a learning community. Over time the students became more conscious of the problem solving process and became more skillful at evaluating the processes available to determine the solutions. This was evident in Socratic questioning from the students. At first students were unaware of how to appropriately ask a good question or how to point out errors in explanations with questions, but over time the students became more familiar with the process. The focus on justifying solutions throughout the treatment seemed to aid in student questioning as well as student confidence in problem solving ability. These factors are important aspects of becoming critical thinking members of society as well as better achievers in math and science. [3]

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[3] Another significant nurturant result from a study such as this is that educators can benefit from reviewing the literature in their content area. See, for example, the accompanying paper on theoretical background. The literature on proportional reasoning provides a wealth of information concerning its importance in cognitive development and its necessity for developing advanced math and science reasoning abilities. The literature also provides valuable information on instructional implications. Awareness and consideration of these concerns can serve as an impetus for change in classroom instruction and curriculum design for proportional reasoning.

IV. STRENGTHS AND LIMITATIONS OF ASSESSMENT

A. Threats to validity

Several factors threaten external validity. The small sample size and lack of diversity in the population are two factors that could not be controlled. The major threat to external validity was the duration of the study. The study was completed for only one unit of instruction occurring over a two week time period.

There are a variety of threats to internal validity, the most significant threat being that two different instructors were involved in the study. Differences in personality, teaching style, expectations, and familiarity with the students could play a major role in the outcomes of the study. Another major threat to validity is the test instrument. Although the test items were taken from examples in the literature, it was not face validated for the specific purpose of this study. This could expose a potential bias for the method of instruction used in the treatment. The chance that students had been exposed to a proportionality algorithm before the instruction began could have an impact on internal validity, as could the variance in times of instruction between the experimental and control group students, and the placement in a classroom allocated for inclusion. In addition, transition students from other districts in both groups and prior experiences in math and science courses could be contributing factors.

B. Strengths of assessment: justification of error rejection and other data implications

A Type I error is a statistical term that describes whether or not the effects of the research occured by chance. The smaller the p value, the less likely that effects are occurring by chance. For instance, the control group basically showed no gain but the experimental group had significantly greater gains, giving weight that this did not occur by chance.

A type II error is a statistical term that describes a situation for which an effect is statistically significant but the researchers really don't think there was a gain. In my research, I argue against this case because I have seen the effect of Modeling Instruction in physics and there is strong evidence for it. Secondly, the control group was a very similar group of students in all categories. If however, the experimental group was a regular class and the control group was 50% special needs students, then the chances of a type II error would be much more likely. In that case, I would not feel confident in saying that the gains were strictly due to instruction.

The rejection of the type I error for the pretest to posttest analysis for the control group is evident in the literature (see the accompanying paper, available from the author). The literature indicates that traditional math instruction does not develop reasoning strategies. In addition, *the literature states that the teaching of the cross product algorithm (i.e., cross-multiplying) actually inhibits proportional reasoning ability. This can be justified by the small to negative gains from pretest to posttest in the control group*. In some cases these students applied an appropriate reasoning strategy before instruction but the cross product algorithm approach was used with blind faith during the posttest. Students either set the algorithm up incorrectly or applied it in situations where it was not applicable. Furthermore, students accepted the solution without even considering what the solution should be with qualitative directional reasoning.

Another literature implication that supports rejection of the type I error for the control group centers around socio-cultural factors. The traditional math instructor more than likely did not emphasize the value of qualitative directional reasoning or justification of solutions through clear explanations and alternative approaches.

Although the scoring system punished students who used an algorithmic approach, the success of the experimental group cannot be attributed to this factor. It was determined early in the study that students in the experimental group had been exposed to the cross multiplication algorithm in prior courses or earlier in the Math I course. Experimental students using this approach were punished in accordance with the scoring system unless an appropriate written explanation was provided indicating proper reasoning. A possible explanation for why students using an algorithm in the experimental group had more success is that they used appropriate directional reasoning first, ensuring they had set the algorithm up correctly.

Type II errors for the experimental group can be rejected for a variety of reasons. The students were presented with problems rather than approaches first. This ensured that students were actively engaged in the problem solving process while determining solutions. Students were required to present these approaches for the class to evaluate. This encouraged students to seek alternative solutions to build defense and support for their claims. This process helped build cognitive frameworks for proportional reasoning through continuous evaluation. The students were also encouraged to use qualitative directional reasoning before computation. Most problems asked for a qualitative prediction before computation, more than likely making this thinking strategy part of their problem solving repertoire. Finally, students in the experimental group received more hands-on experiences, helping make unfamiliar situations on the posttest more comfortable.

V. IMPLICATIONS FOR EFFECTIVE TEACHING AND LEARNING

The need for this study stemmed from awareness of student performance on the TIMMS by students in the United States in addition to observations made throughout the teaching career of the author. The literature described many aspects of curriculum and instruction that could be underlying factors for the observations and performances. This study made use of many recommendations from the literature concerning curriculum and instruction in proportional reasoning and incorporated them with Modeling Instruction, a successful model for teaching physics. The results of this study suggest that modeling methods in conjunction with instructional implications from the proportional reasoning literature do in fact have a significant effect on attaining proportional reasoning ability. These instructional methods could lead to better preparation for secondary science courses, higher achievement on international exams, and possibly serve as an instructional approach for developing formal operational thinking. These implications are important; and in fact many of these implications for effective teaching and learning are beneficial even if the results of the study were not significant.

The modeling methods used place an emphasis on higher order thinking skills. Requiring students to approach problems without prior instruction makes it necessary for students to incorporate prior knowledge and skill into the problem solving process. Students must be able to search for patterns and reasoning constructs that enable them to be successful on problems that they have not encountered. This shifts their focus away from being answer centered, so they rely on reasoning to evaluate their process. The presentation and Socratic questioning during whiteboarding require them to continually evaluate and modify their reasoning strategy. This evaluative process allows students to find a strategy that is typically the best alternative for a solution.

The whiteboarding presentation also creates an atmosphere that is centered around learning. After a minimum comfort level is established, students begin to appreciate the diversity and cleverness with which they can determine solutions. Students often learn from incorrect solutions, and they develop a genuine respect for being able to recognize problems with solutions. This provides students with the experience of seeing incorrect strategies, and it results in eliminating them from their repertoire of possible alternatives. If a solution cannot be determined, then classroom discourse continues until possible approaches are developed. This provides students with confidence and helps validate correct solution strategies while providing them with the power of learning at the same time.

Requiring students to focus on the problem solving process for classroom presentation makes them more accountable for the reasoning and skills that are to be developed through course objectives. Students can no longer copy down answers from classmates or slide through the course without their lack of learning being noticed by the instructor and their peers. All students present at one time or another, and because the focus is on the process, a student cannot simply state "this is what I got for an answer". They must explain their process and justify why their solution is valid. This process is not for humiliation; it is simply putting the students in a position where they are held accountable for what they are learning. Most students recognize this aspect and thus put more effort into understanding why a particular process or skill is necessary to produce a solution.

The process of whiteboarding also provides students with experience in building defense and support for their arguments. Since students must justify their solutions, they must construct an argument for support through the particular reasoning strategy that they have chosen to obtain a solution. Because they must support their argument, students are much more likely to show all of the work necessary to solve a particular problem in addition to providing appropriate units and labels for the numbers in the problem. This allows students to use mathematics as a language for expressing their reasoning rather than view it as an isolated set of rules to get an answer.

The modeling methods also develop skills in the instructor. The presentation of student solutions makes students' thinking visible to the instructor. This is compounded by the student discourse that occurs in the room along with any presentation. The instructor becomes very aware of how individual students are thinking, not only by the presentations themselves but by the questions other students ask the presenters. The process allows the instructor to constantly monitor where the students are at with a particular skill or reasoning construct. In effect, instructors can become their own diagnosticians. From student discourse, the instructor can determine what types of problems can be attempted next or determine if additional problems are needed to clarify the skills and reasoning patterns that serve as the best alternative for a particular problem context. The role of the instructor then focuses on choosing appropriate learning activities, guiding students through discourse, and evaluating when the next step in the learning process should take place.

The final implication is that hands-on experiences not only help with unfamiliar experiences but also enhance learning experiences. The hands-on activities in this study provided a platform for real problems that were related to proportional reasoning. Using a hands-on approach made the proportional reasoning teaching and learning construct more concrete in nature. The concrete nature helped make the situations more tangible. These tangible situations serve as a foundation that can be referred back to, when evaluating a reasoning strategy, in addition to providing opportunities for students to test and evaluate their claims. The ability to hypothesize and test claims for these situations proved to be a valuable experience for students because it reinforced their reasoning strategies or provided evidence that they did not work.

These implications serve as a platform for true learning. Anyone can mimic a procedure from an instructor and gain little from it. Although it may be hard to describe and conceive, direct experience with modeling methods and the rich learning experiences that come with it are convincing that true learning is taking place.

VI. IMPLICATIONS FOR FUTURE RESEARCH

Proportional reasoning is said to be at the pinnacle of cognitive development with its relation to formal operational thought. In addition, proportional reasoning is recognized as a fundamental reasoning construct necessary for mathematics and science achievement. Cognitive development, methodology, instructional implications, and relationships to higher achievement in math and science have all been considered in this study. Although this study had significant results, its short duration along with other factors that may pose threats to validity produced many questions.

1. Can using the Modeling Method of instruction with suggestions from proportional reasoning research produce long-term effects that are evident in advanced high school science courses?
2. Can Modeling Instruction strategies combined with proportional reasoning research results facilitate development of formal operational thinking?
3. What specific components of Modeling Instruction contribute to development of formal operational thinking? Must modeling strategies be used or are interactive engagement methods (minds-on and usually hands-on, with immediate feedback) sufficient, without

focusing on models and mathematical modeling?
4. Will Modeling Instruction (or interactive engagement without modeling strategies) used with research findings in proportional reasoning produce similar effects when sociocultural and socioeconomic factors are considered?
5. Can modeling methods or other varieties of interactive engagement help change sociocultural factors that affect math and science instruction?
6. Can modeling methodology help develop a unified vision for mathematics and science in education?
7. Will students become more effective at thinking and reasoning if modeling methods are employed consistently in math and science through the duration of a student's education?
8. What types of curriculum changes are necessary for consistent use of modeling methodology in K-12 math and science?

Many other questions could arise as Modeling Instruction and other varieties of interactive engagement continue to develop and gain support. It is also necessary to consider the continually developing fields of cognitive science and research in proportional reasoning. As all these fields develop, answers to these questions may emerge. For the time being, it is necessary to respect the complex nature of these factors that affect student learning. If the goal of education is to develop members of society who function at a high thinking level, then the ideas presented in this paper along with the questions raised may serve as an initial frame of reference for continuing research.

ACKNOWLEDGMENTS

I am grateful to Richard Hake, Emeritus Professor of Physics at Indiana University, and Jane Jackson, Co-Director of the Modeling Instruction Program at Arizona State University, for substantive and editorial suggestions. This article describes work done for a Master's thesis in the School of Education at Drake University in Des Moines, Iowa.

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APPENDIX A: definitions

Concrete operational. Capable of processing information that can be physically manipulated and experienced.

Constructivist. Refers to the philosophy that supports the process of a student actively constructing knowledge in personally meaningful ways.

Disequilibrium. When a child encounters a situation that s/he has never experienced before, and/or his/her old ways of thinking are challenged, the child enters into a state of disequilibrium, or a state of intellectual conflict.

Formal operational. "Meaning is given to these concepts not through the senses but through imagination or through logical relationships within the system." (Lawson, 1975,p.348)

Interactive engagement methods. Operationally defined as "those designed at least in part to promote conceptual understanding through interactive engagement of students in heads-on (always) and hand-on (usually) activities which yield immediate feedback through discussion with peers and/or instructors", as opposed to "relying primarily on passive-student lectures, recipe labs, and algorithmic-problem exams" (Hake, 1998a,b).

Manipulative approach. The manipulations of numbers and variables through a specified process or set of rules. Usually used in conjunction with an algorithm and differing from a hands-on approach where objects, often referred to as manipulatives, are used to make the experience more concrete in nature.

Mathematical modeling. The systematic development of mathematical reasoning abilities through recognition of patterns between numerical data, graphical relationships, equations, and evaluation techniques that require use of mathematics as a language to establish relationships.

Metacognitive. Thinking about thinking.

Reasoning pattern. An identifiable and reproducible thought process directed at a type of task.

Rote. Learning strictly through abstract memorization.

Solved. Giving the right answer justified by the right interpretation.

Traditionalism. The type of learning philosophy that has been the norm for 100 years.

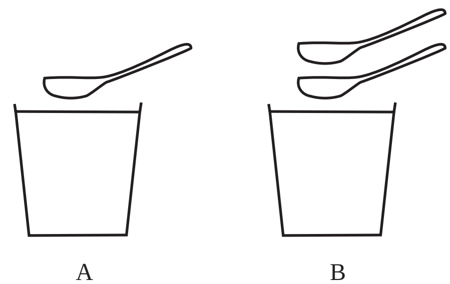
Whiteboarding. The process of using hand held marker boards to present solutions that are to be verbally justified by the presenter in addition to serving as a platform for classroom discourse.

APPENDIX B: Proportional Reasoning Diagnostic Test

A solution is considered correct only if it includes the correct explanation for how that answer was obtained. Please show all work for obtaining your solution.

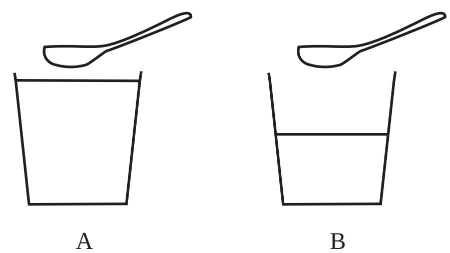
1. Cup A and cup B are both full of water. To cup A one spoonful of sugar was added, and to cup B two spoonfuls of sugar were added. Which of the cups is sweeter, or is the sweetness in both cups the same?

Explain:



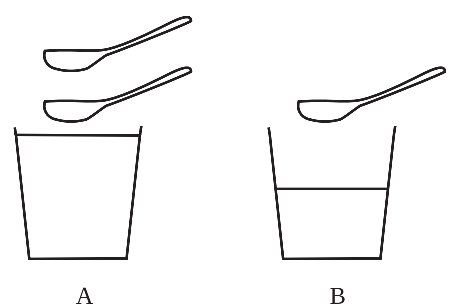
2. Cup A is *full* of water. Cup B is *half* full of water. To cup A one spoonful of sugar was added and to cup B one spoonful of sugar was added. Is one of the cups sweeter than the other, or are they of the same sweetness?

Explain:



3. Cup A is *full* of water, and cup B is *half* full of water. To cup A two spoonfuls of sugar were added and to cup B one spoonful of sugar was added. Is one of these cups sweeter than the other, or are they of the same sweetness?

Explain:



4. If today you mix *less* lemonade mix with *more* water than you did yesterday, your lemonade drink will taste:

a. stronger b. weaker c. exactly the same d. not enough information is provided.

Explain:

5. You and your friend hammer a line of nails into different boards. You hammer more nails than your friend. Your board is shorter than your friend's. On which board are the nails hammered closer together?

a. your board b. your friend's board c. the nails are equally spaced. d. not enough information is provided.

Explain:

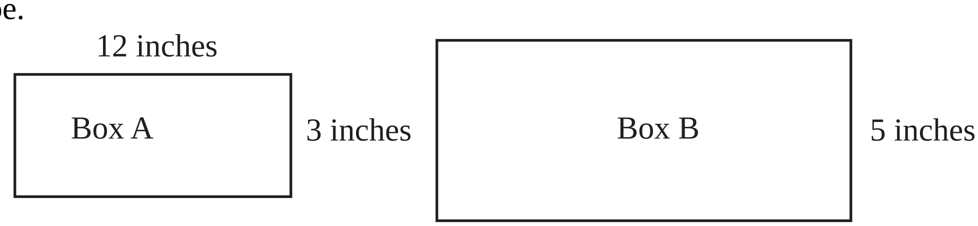
7. You and your friend are using different road maps of a city. On your map a road 3 inches long is really 15 miles long. On your friend's map, a road 9 inches long is really 45 miles long. Who is using a larger city map?

a. you b. your friend c. both maps are the same. d. not enough information is provided.

Explain:

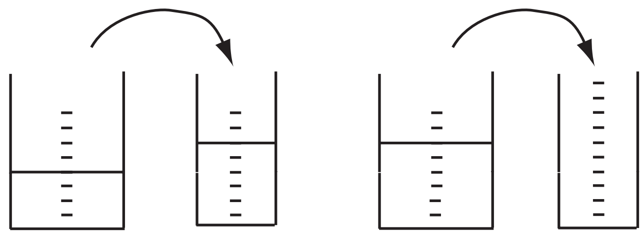
6. You and your friend drove equally fast along a country road. It took you 6 minutes to drive 4 miles. How long did it take your friend to drive 6 miles? Show your reasoning and justify your answer in words.

8. Box A has the following dimensions. Determine the length of box B in order to make it the same shape.



Show your reasoning and justify your answer in words.

9. You pour water into a wide cylinder up to the fourth mark (figure A). Then you transfer this water into the narrow cylinder, and the water rises to the sixth mark (figure B).



A B C D

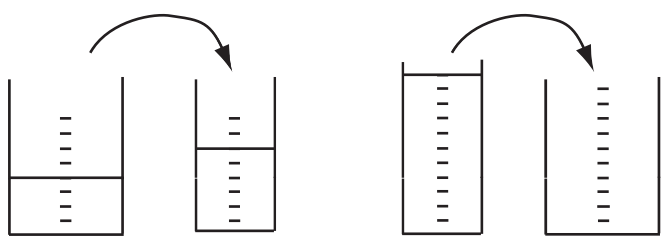
What if you started instead with water in the wide cylinder at the *6th* mark (figure C); how high would the water rise if you transferred it into the narrow cylinder (figure D)?

a. 7 b. 8 c. 9 d. 10 e. other f. cannot predict

Explain, using math and words.

10. You pour water into a wide cylinder up to the 4th mark (figure A). You then transfer this water into the narrow cylinder, and the water rises up to the 6th mark (figure B).

If instead you poured water into the *narrow* cylinder up to the11th mark (figure C), how high would the water rise if you transferred it into the *wide* cylinder (figure D)?

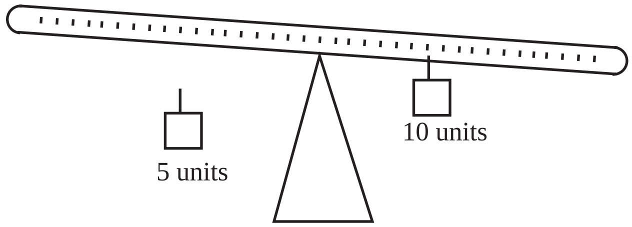


A B C D

a. 5-1/3 b. 5-2/3 c. 7-1/3 d. 7-1/2 e. 8 f. 8-1/2 g. 9 h. other i. cannot predict

Explain, using math and words.

11. A balance beam has two hanging weights. You hang a 10-unit weight 7 marks from the center, on the right.

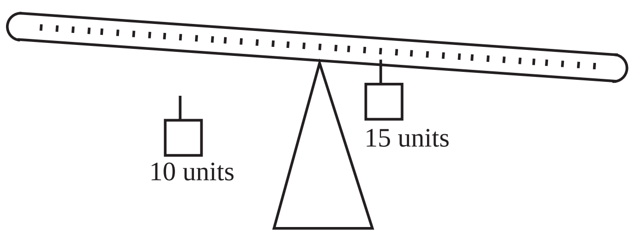


Where would you hang the 5-unit weight on the left side, to make the beam balance?

a. between marks 3 & 4   b. mark 7   c. mark 12   d. mark 14   e. at the end

Explain, using math and words.

12.   A balance beam with two different weights is now used. You hang a 15-unit weight 4 marks from the center, on the right.



Where would you hang a 10-unit weight on the left, to balance the beam?

a. mark 5   b. mark 6   c. between marks 6 & 7   d. mark 7   e. mark 8   f. mark 9
g. mark 15

Explain, using math and words.

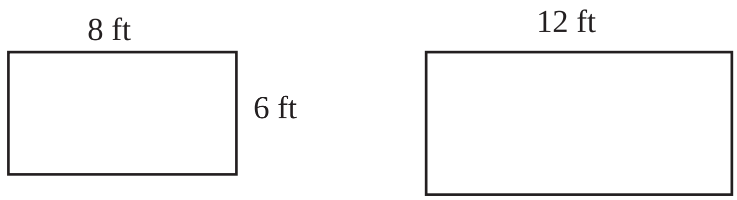
APPENDIX C: classroom assignments

Proportional reasoning assignment II:

1. A building company has been requested to build between 35 and 45 apartments. Each time they build three 1-bedroom apartments, they must build four 2-bedroom apartments, and one 3-bedroom apartment. What total number of apartments must they build, and how many of each type of apartment will there be?

2. Seven girls have three pizzas, and three boys have one pizza. Who gets to eat more pizza: each boy or each girl?

3. Calculate the unknown width of the rectangle in order to make it look the same shape, only larger.



Proportional reasoning assignment III:

Answer the following questions by completing the following for each.

a. Make a prediction for the missing value of more, less, or equal for each of the problems.
b. Determine a solution to the problem.
c. Explain your solution or show your work. Put labels on your numbers and show all steps.
d. Prove your solution by setting up ratios and comparing them to each other.

1. A farmer has 12 hens. They lay a combination of white and brown eggs. For every 9 white eggs there are 3 brown eggs. How many brown eggs should the farmer expect, if the average number of eggs laid each day is 144?

2. The cost of a TV commercial is $100,000 for every 30 seconds. How much should a company expect to pay for a 45 second commercial?

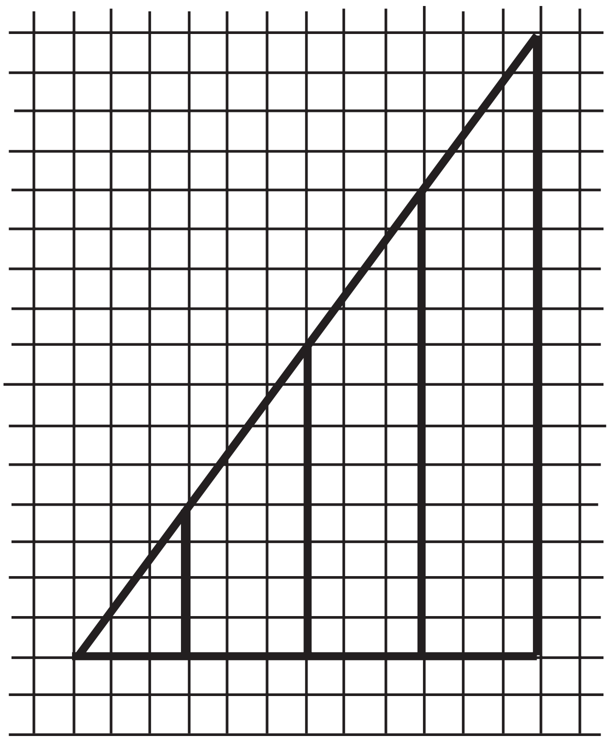
3. A baseball team won 15 of its first 23 games. How many games would you expect them to win in a 112 game season if they perform at the same level?

4. If it takes 3 painters 8 hours to paint a house, how much time would you expect 7 painters to paint a house of similar size? Assume the painters work at about the same rate each.

5. If a 15-inch pizza costs $10.99, how much would you expect to pay for a 5-inch personal pan pizza, assuming the restaurant uses the same pricing strategy for all pizzas?

Proportional reasoning assignment IV: (Put all solutions on the back)

1. Look at the graph below. How many triangles do you see? Explain your choice.



For the following questions we will be looking at the base and the height of the triangle(s). They are the two sides that form the 90-degree angle with each other. Consider each square's height and width on the graph to represent one unit of length.

2. How are the two sides of the triangles(s) related to each other? Explain your reasoning, and if there is more than one way, please explain all methods.

3. Can you predict the length of the sides for a fifth and sixth triangle if they were drawn on the paper? Report the base and height, and explain how you figured it out.

4. Use ratios to prove whether or not all of the triangles are of equivalent proportions, and explain.

5. If the base of a triangle is 1.5 units, what would the height be? Explain how you figured it out.

6. What would the height of a triangle be, if the base were 22.5 units? Explain your reasoning.

7. Use the reasoning that you used to solve the above problems to solve the following problem. Suppose you are 5.5 feet tall. You are trying to predict the height of a building. You cannot directly measure the height of the building but you are standing in its shadow. You measure the length of the building's shadow and find that it is 25 feet long. Your shadow is 3.5 feet tall.

a. Diagram the situation and show where you would have to stand and how the shadows would be arranged in order to make the problem similar to the ones you did above.
b. Use the information given and the process learned above to determine the height of the building in feet. Explain how you got your answer.