Inductively Modeling Parallel, Normal, and Frictional Forces

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This year, instead of resolving the weight *mg* of an object resting on an incline into force components parallel and perpendicular to the surface of the incline, I asked my students to actually measure these forces at various angles of inclination and graph the data. I wanted my students to inductively discover $mg \sin \theta$ and $mg \cos \theta$, and to use these graphs to confront the passive nature of the static frictional force.[1] I believe the graphs themselves are very powerful conceptual tools that are often never discovered and used by students who only learn to use equations at specific angles to solve specific quantitative problems.

I started this project by instructing my students to use Vernier[2] dual-range force sensors set at the ±10-N range to measure the magnitude of the force parallel to the surface of a dynamics track acting on low- friction dynamics carts (mass = 0.500 ± 0.005 kg) as a function of the angle of inclination of the tracks. The students started with their dynamics tracks at 0° relative to the floor and changed the angle of inclination by 10° increments. We assumed that the static frictional forces within the carts' wheels and between the carts' wheels and the tracks were negligible, so that we were essentially measuring a gravitational force, or rather a component of it. Next we used a Vernier[2] force plate set at the -200/+850-N range to measure the normal force acting on a flat, circular weightlifting plate labeled with a mass of 11.3 kg that we borrowed from the weightroom. We used the Force Plate to measure the normal force because it has a large, flat surface (30 cm by 28 cm), which makes it ideally suited for making normal force measurements. I chose the 11.3-kg weightlifting plate because it was easy to work with, and because of its significant mass the normal force would still be measurable and significant at large angles such as 80°. Again we started at 0° and approached the vertical by changing the angle of inclination of the force plate by 10° increments. Table I contains all the data collected by a typical student lab group.

Table I. Parallel (P) and Normal (N) force data measured by student lab group.

Angle (degrees)	P Force (N)	N Force (N)	sin (angle)	cos (angle)
0	0.000	111.0	0.00000	1.00000
10	0.801	109.0	0.17365	0.98481
20	1.499	102.2	0.34202	0.93969
30	2.274	93.4	0.50000	0.86603
40	3.008	81.7	0.64279	0.76604
50	3.729	71.5	0.76604	0.64279
60	4.210	55.4	0.86603	0.50000
70	4.601	40.5	0.93969	0.34202
80	4.986	23.5	0.98481	0.17365
90	5.060	0.0	1.00000	0.00000

Then I coached the students in their graphical analyses of the magnitudes of the parallel and normal forces acting on the dynamics carts and weightlifting plate. I started by instructing the students to plot the magnitudes of the parallel and normal forces as

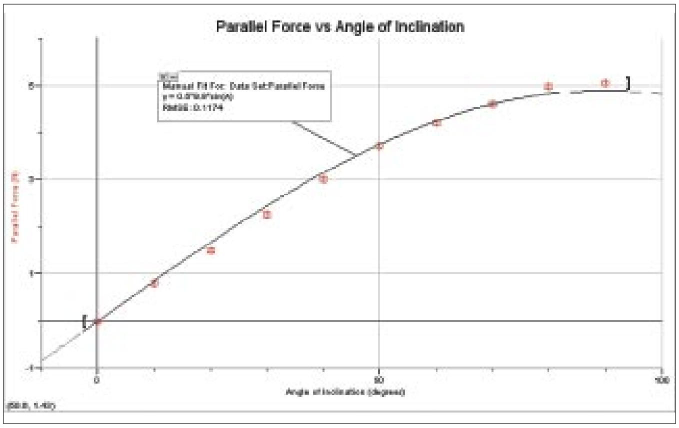

Fig. 1. Student graph showing the magnitude of the gravitational force acting on a low-friction dynamics cart parallel to the dynamics track the cart is resting on as a function of the angle of inclination of the track. Students manually curve-fit mg sin θ to their data.

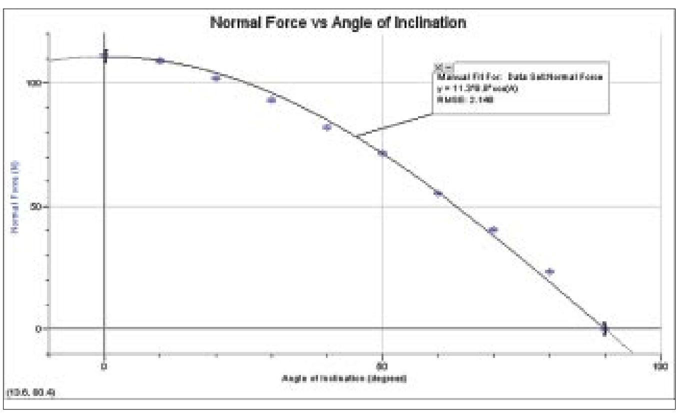

Fig. 2. Student graph showing the magnitude of the normal force acting on an 11.3-kg weightlifting plate resting on a force plate as a function of the angle of inclination of the force plate. Students manually curve-fit mg cos θ to their data.

a function of the angle of inclination. This produces two curves. One of the curves strongly resembles a sine function, and the other curve strongly resembles a cosine function (see Figs. 1 and 2). Next I asked my students to straighten the curves by graphing the appropriate functional relationships. I was somewhat surprised when none of the students tried the trigonometric functions, but unfortunately most students have had very little experience with graphical analysis. Therefore, I coached the students toward the trigonometric functions by offering this guiding question: What elementary functions use angles as inputs? This small nudge was all the students needed, for they quickly straightened the curves by graphing the magnitude of the parallel force as a function of the sine of the angle of inclination and by graphing the magnitude of the normal force as a function of the cosine of the angle of inclination (see Figs. 3 and 4). Finally, I asked the students to interpret the physical meaning of the slopes of their straight lines (*mg* the weight of the object), and once this was accomplished the students have inductively discovered $mg \sin \theta$ and $mg \cos \theta$. The students then validated their mathematical models by manually curve-fitting $mg \sin \theta$ and $mg \cos \theta$ to their data (see Figs. 1 and 2). I congratulated the students on their discovery.

A Couple of Experimental Notes

Make sure the students zero the force sensors each time they make a new force measurement at a new angle of inclination. To measure the normal force, I C-clamped a thin board at each end across a short, sturdy board wide enough to support the force plate to prevent it from slipping down that board. When you rest the force plate up against this thin board, make sure that the surface of the force plate does not touch the thin board. I also attached a C-clamp to the top of the board supporting the force plate, and attached a string to this C-clamp and the weightlifting plate resting on the force plate to prevent the weightlifting plate from sliding. To prevent the board supporting the force plate from slipping along the floor, we used a carpeted floor to provide enough friction. Also, an electronic balance can be used to measure the magnitude of relatively small normal forces.[3]

Understanding the Passive Nature of the Static Frictional Force

The next day in class I showed the students a small inclined plane with a hockey puck resting on it. I tilted the plane slowly upward until I reached the approximate angle of repose. If I tilt the plane at a greater angle, the puck will slide and accelerate down the plane. I asked the students to use their whiteboards to explain the motion of the puck on the inclined plane for any angle of inclination from 0° to 90° and to present their ideas for review by the entire class. I offered the students one suggestion: Sketch graphs for the absolute values of the relative magnitudes of the forces acting on the puck parallel to the plane as a function of the angle of inclination, and put all the graphs on one set of axes. Then I divided the class into small groups of three or four students. Each group took an inclined plane, a hockey puck, and a whiteboard back to their table to begin connecting the concepts they had already learned to this new problem.

The students typically start by sketching an increasing concave down curve that represents the relative magnitude of the force parallel and down the plane acting on the puck—$mg \sin \theta$. I often ask the students for the origin of this force. Hopefully, the students will respond with the gravitational interaction between the Earth and the puck. The students then start thinking about the magnitude of the frictional force between the surfaces of the plane and the puck opposing the relative motion of the puck at different angles of inclination. The students have already used force sensors to measure the frictional forces between two horizontal surfaces and understand that the magnitude of the maximum static frictional force is directly proportional to the normal force. I coach the students to use this relationship and expand it to encompass the more general case where the angle of inclination for the two surfaces may not be zero. Most groups then sketch a decreasing concave down curve that represents the relative magnitude of the maximum static frictional force acting on the puck up the plane—$\mu_s mg \cos \theta$, where μ_s represents the coefficient of static friction. The students now should have two curves that intersect at a particular angle (see Fig. 5), which I will refer to subsequently as θ_i—the angle of intersection.

I guided the students by suggesting that they focus their discussions around what θ_i is trying to tell them about the motion of the puck. This usually causes some conceptual conflicts, which is just what I wanted. When the students look at the two curves, it is clear that the magnitude of the maximum static frictional force acting on the puck up the plane is greater than the magnitude of the gravitational force acting on the puck down the plane from 0° to just before θ_i. However, the students have observed that the puck does not ever accelerate up the plane. At this point many students came to me and asked for help. I coached the students by posing this question: If the puck does not move on your inclined plane, can the magnitude of the static frictional force actually be equal to the value predicted by your curve?

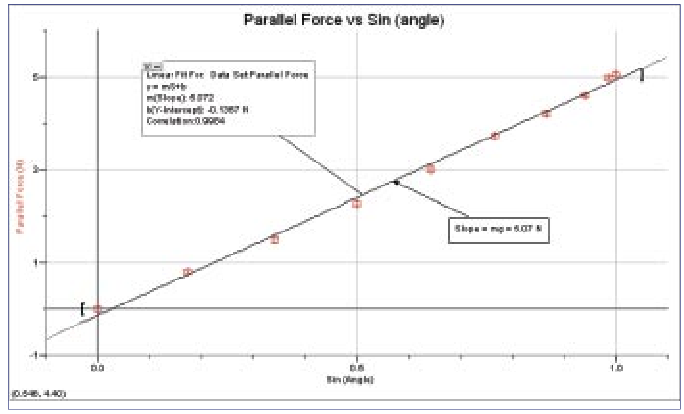

Fig. 3. Student graph showing the magnitude of the gravitational force acting on a low-friction dynamics cart parallel to the dynamics track the cart is resting on as a function of the sine of the angle of inclination of the track. Students fit a straight line to their data and discovered that the slope of the line was approximately equal to the weight of their dynamics cart.

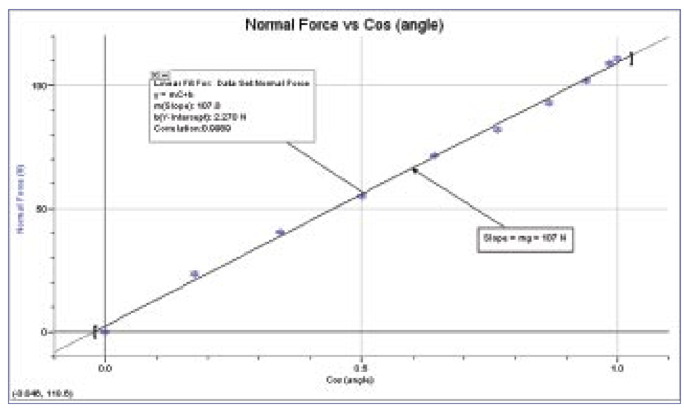

Fig. 4. Student graph showing the magnitude of the normal force acting on an 11.3-kg weightlifting plate resting on a force plate as a function of the cosine of the angle of inclination of the force plate. Students fit a straight line to their data and discovered that the slope of the line was approximately equal to the weight of the 11.3-kg weightlifting plate.

The students must now confront the passive nature of the static frictional force—a force that adjusts its magnitude to just balance the magnitude of an applied force up to a maximum value at which sliding begins. This passive nature of the static frictional force is usually represented mathematically by the interval expression ($0 \leq f_s \leq \mu_s F_N$) where f_s represents the force of static friction, F_N represents the normal force, and $\mu_s F_N$ represents the maximum force of static friction. Most groups now realize that the actual

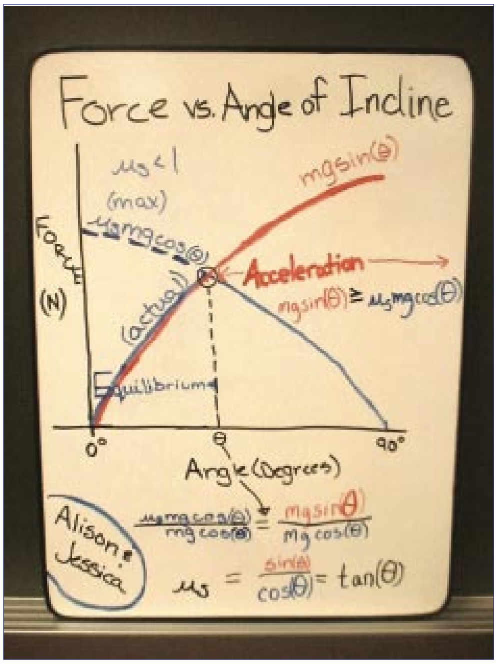

Fig. 5. Students' whiteboard explanation of the motion of a hockey puck resting on an inclined plane tilted from 0º to 90º.

magnitude of the static frictional force acting on the puck from 0° to θ_i is just equal to the component of the gravitational force acting on the puck parallel to the plane—$mg \sin \theta$. This is a true moment of physics insight for most students and one that they do not forget. The expression $\mu_s mg \cos \theta$ represents the magnitude of the *maximum* static frictional force and is equal to the static frictional force acting on the puck at the moment of release for any angle θ greater than θ_i. Now the students can easily explain the motion of the puck. To the left of θ_i, the puck is in static equilibrium because the magnitude of the static frictional force acting on the puck up the plane is adjusting to just balance the magnitude of the gravitational force acting on the puck down the plane. At θ_i, and only at θ_i, the magnitude of the maximum static frictional force acting on the puck up the plane is equal to the component of the gravitational force acting on the puck down the plane, so $\mu_s mg \cos \theta = mg \sin \theta$ at this angle only. When the angle of the incline is greater than θ_i (and only at the instant that the puck is released), $mg \sin \theta > \mu_s mg \cos \theta$; the puck then begins to accelerate down the incline (see Fig. 5). It is important to note that at the instant the puck begins to slide down the incline, the frictional force decreases slightly because it is transitioning from static to kinetic friction.

Conclusion

It clearly takes a couple of lab periods to help students inductively discover $mg \sin \theta$, $mg \cos \theta$, and to understand the passive nature of the static frictional force ($0 \leq f_s \leq \mu_s F_N$). However, I am convinced by my students' whiteboard explanations and presentations that it is time well spent by both the instructor and the student. I adapted the pedagogy for this project from the modeling methodology developed by Hestenes[4,5] and others at Arizona State University.

References:

1. A.B. Arons, *Teaching Introductory Physics* (Wiley, New York, 1997), p. 76.
2. The Vernier Dual-Range Force Sensor, Order Code DFS-BTA, price $109. The Vernier Force Plate, Order Code FP-BTA, price $199. Vernier Software & Technology, 13979 S.W. Millikan Way, Beaverton, OR 97005-2886; http://www.vernier.com.
3. Jonathan Mitschele and Matthew Muscato, "Demonstrating normal forces with an electronic balance," *Phys. Teach.* **32**, 555–556 (Dec. 1994).
4. Malcolm Wells, David Hestenes, and Gregg Swackhamer, "A modeling method for high school physics instruction," *Am. J. Phys*. **63**, 606 (July 1995).
5. Many excellent papers on modeling may be downloaded from the Arizona State University website at http://modeling.la.asu.edu/modeling-HS.html. Navigate to Key Articles by Modeling Teachers.

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